Tutorial CFD
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Tutorial 3

Submission: Friday, 23.05.03, 12.00, metal box "Rutka", the ground floor, building 48.
Download: www.mathematik.uni-kl.de/~rutka/CFD_tutorial.html

Exercise 1

Steady unidirectional flow of a viscous incompressible fluid takes place in between two infinitely long concentric circular cylinders under a given pressure gradient P. Solve the Navier Stokes equation for this case and obtain the velocity field. Determine the volume flux across a cross section. At the walls of the cylinders, there is no-slip boundary condition.

Exercise 2

Solve the problem of steady unidirectional viscous flow through an infinitely long pipe of elliptical cross section. Also find the volume flux across a cross section.

Hint: Take the equation of ellipse as

$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} - 1 = 0\,.$$

The no-slip boundary condition will help you guess the solution easily!

Exercise 3

The rate of change of the angular momentum of the material volume $V(t)$ with surface $S(t)$ is equal to the moment of the forces acting on it:

$$\frac{D}{Dt}\int_{V(t)} \underline{r} \times \rho\underline{u}\,\mathrm{d}V = \int_{V(t)} \underline{r} \times \rho\underline{f}\,\mathrm{d}V + \int_{S(t)} \underline{r} \times \underline{\tau}\,\mathrm{d}S\,,$$

where $\underline{r} = x_1 i + x_2 j + x_3 k$ is the position vector, $\underline{f}$ is the external force per unit mass and $\underline{\tau}$ the stress vector on $S(t)$.

Show that this leads to symmetry of the stress tensor, i.e., $T_{ij} = T_{ji}$.

Hint: Use transport theorem and the linear momentum conservation law.