Tutorial CFD
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Tutorial 4

To be discussed on: Monday, 2.06.2003.
Download: www.mathematik.uni-kl.de/˜rutka/CFD_tutorial.html

Programming Exercise

Consider linear advection equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

with a **periodic** initial condition, given by
A)

$$u_0(x) = \begin{cases} x & , \quad 0 \le x \le 1/2 \\ 1-x & , \quad 1/2 \le x \le 1 \end{cases}$$

B)

$$u_0(x) = \begin{cases} 1 & , \quad -5 \le x \le 0 \\ -1 & , \quad 0 \le x \le 2 \end{cases}$$

Implement the following schemes ($\lambda = k/h$, k — time step, h — spatial step):

1. Upward differences
$$u_j^{n+1} = u_j^n - a\lambda(u_{j+1}^n - u_j^n)$$
2. Downward differences
$$u_j^{n+1} = u_j^n - a\lambda(u_j^n - u_{j-1}^n)$$
3. Lax–Wendroff
$$u_j^{n+1} = u_j^n - \frac{a\lambda}{2}(u_{j+1}^n - u_{j-1}^n) + \frac{a^2\lambda^2}{2}(u_{j+1}^n - 2u_j^n + u_{j-1}^n)$$
4. Lax–Friedrich
$$u_j^{n+1} = \frac{1}{2}(u_{j+1}^n + u_{j-1}^n) - \frac{a\lambda}{2}(u_{j+1}^n - u_{j-1}^n)$$

Choose k and h s.t. the CFL condition is satisfied. Investigate what happens if it is not satisfied.

Choose different values for a. (Take 1, -1, 100, -100). What happens?

Note: Be careful with approximation of boundary conditions!