Gradient Flow Scheme for Nonlinear Fourth Order Equations

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Abstract. Evolution equations with an underlying gradient flow structure have since long been of special interest in analysis and mathematical physics. In particular, transport equations that allow for a variational formulation with respect to the L^2-Wasserstein metric have attracted a lot of attention recently. The gradient flow formulation gives rise to a natural semi-discretization in time of the evolution by means of the minimizing movement scheme (see, e.g. [1]), which constitutes a time-discrete minimization problem for the (sum of kinetic and potential) energy. On the other hand, nonlinear diffusion equations of fourth (and higher) order have become increasingly important in pure and applied mathematics. Many of them have been interpreted as gradient flows with respect to some metric structure.

When it comes to solve equations of a gradient flow type numerically, it is natural to ask for appropriate schemes that respect the equation's special structure in some way. We propose a fully discrete variant of the minimizing movement scheme for numerical solution of the nonlinear fourth order Derrida-Lebowitz-Speer-Spohn equation (cf. [2]). Our method is iterative and consists of an outer and an inner loop. In each time step of the outer loop a constrained quadratic optimization problem for the Fisher information is solved on a finite-dimensional space of ansatz functions. These subproblems are solved iteratively in the inner loop by applying Newton's method to the optimality system, leading to a sequential quadratic programming method. In result, we obtain a fully practical numerical scheme for a non-linear fourth order equation that respects its Wasserstein gradient flow structure.

Joint work with Daniel Matthes and Pina Milisic, see [3].

References:

[1] L. Ambrosio and G. Savaré. Gradient flows of probability measures. In: Handbook of Evolution Equations, Dafermos, C. and Feireisl, E. (eds.), Vol. 3, pp. 1-136, Elsevier, 2006.

[2] A. Jüngel and D. Matthes. The Derrida-Lebowitz-Speer-Spohn equation: existence, nonuniqueness, and decay rates of the solutions. SIAM J. Math. Anal. 39(6), 1996-2015, 2008.

[3] B. Düring, D. Matthes, and J.-P. Milisic. A gradient flow scheme for non-linear fourth order equations. Discrete Contin. Dyn. Syst. Ser. B 14(3) (2010), 935-959.