Reduced Basis Methods and A Posteriori Error Estimation for Saddle Point Problems in Parametrized Domains

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Abstract. The reduced basis (RB) method is a model-order reduction approach which permits the efficient yet reliable approximation of input-output relationships induced by parametrized partial differential equations. Its application to saddle point problems such as the Stokes and Navier-Stokes equations in parametrized domains confronts us with several difficulties: the incompressibility condition imposed on the velocity causes not only complications in the choice of the RB approximation spaces, but also in the construction of rigorous and computationally efficient *a posteriori* error bounds.

Existing error bounds (cf. [2]) require the very expensive evaluation of lower bounds to inf-sup stability constants. We pursue a penalty approach (cf. [3]) that enables us to entirely circumvent the computation of inf-sup constants: we obtain computationally very efficient and reasonably sharp error bounds at the expense of an additional error in the velocity and pressure *truth* approximation, on which the RB approximation is built.

Existing error bounds moreover bound the total error in the RB approximation. Since many engineering outputs depend on either the velocity or the pressure solution, we are particularly interested in the development of separate rigorous and efficient *a posteriori* error bounds for the velocity and pressure approximations. For saddle point problems such as the Stokes equations, we derive separate bounds from a stability result by F. Brezzi (cf. [1]), and compare their sharpness, performance, and efficiency with those of existing bounds.

[1] F Brezzi, On the existence, uniqueness and approximation of saddle-point problems arising from lagrangian multipliers, *R.A.I.R.O.* 8 II, 129-151, 1974.
[2] S Deparis, G Rozza, Reduced basis method for multi-parameter-dependent steady Navier-Stokes equations: application to natural convection in a cavity, *Journal of Computational Physics* 228, 4359-4378, 2009.
[3] A-L Gerner, K Veroy, Reduced basis a posteriori error bounds for the Stokes equations in parametrized domains: a penalty approach, *Mathematical Models and Methods in Applied Sciences (M3AS)*, accepted 2010.