Universität Konstanz Sommersemester 2013
Fachbereich Mathematik und Statistik
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Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 2

Deadline for hand-in: 2013/05/13 at lecture

Exercise 4 (4 Points)
Consider the quadratic function $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle + \gamma$$

where $Q \in \mathbb{R}^{n\times n}$ is symmetric, $c \in \mathbb{R}^n$ and $\gamma \in \mathbb{R}$.

Show directly, i.e. without using any theorem from the scriptum, that the following holds:

$$f \text{ is convex} \Leftrightarrow Q \text{ is positive semidefinite.}$$

Exercise 5
Consider the quadratic function $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle + \gamma$$

where $Q \in \mathbb{R}^{n\times n}$ symmetric and positive definite, $c \in \mathbb{R}^n$ and $\gamma \in \mathbb{R}$. Let $x^k \in \mathbb{R}^n$ be arbitrary and $d^k \in \mathbb{R}^n$ a descent direction of f in x^k for a $k \in \mathbb{N}$.

Find the (exact) step size s^* in direction d^k such that the decreasing of $f(x^k + s^* d^k)$ is maximal.

Exercise 6
Consider the general descent method (as known from the lecture) for the function

$$f : \mathbb{R} \to \mathbb{R}, \qquad f(x) = x^2,$$

with starting point $x^0 := 1$ and the following directions $d^k \in \mathbb{R}$ and step-sizes $t^k \in \mathbb{R}$:

1. $d^k := -1$, $t^k := \left(\frac{1}{2}\right)^{k+2}$ with $k \in \mathbb{N}_0$,
2. $d^k := (-1)^{k+1}$, $t^k := 1 + \frac{3}{2^{k+2}}$ with $k \in \mathbb{N}_0$.

Verify that these choices lead to a decrease of the function f. For that, present the sequence x^k generated by the steepest descent method using induction with respect to k. Further determine in each case the limit $(\lim f(x^k))$ and compare it to the minimum of $f(x)$. Comment on the result!