Universität Konstanz Sommersemester 2013
Fachbereich Mathematik und Statistik
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Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 6

Deadline for hand-in: 2013/07/08 at lecture

Exercise 16 (Preconditioning)
Consider the quadratic function $f : \mathbb{R}^2 \to \mathbb{R}$,

$$f(x,y) = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 100 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 3,$$

with the preconditioning matrices

$$H = \mathrm{Id} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad H = \nabla^2 f = \begin{pmatrix} 100 & -1 \\ -1 & 2 \end{pmatrix}, \quad H = \begin{pmatrix} f_{xx} & 0 \\ 0 & f_{yy} \end{pmatrix} = \begin{pmatrix} 100 & 0 \\ 0 & 2 \end{pmatrix}.$$

Use the Gradient Method you implemented for the first program sheet on $\tilde{f}$ defined on sheet 5, exercise 15, to determine the number of gradient steps required for finding the minimum of f with the different preconditioning matrices H and initial value `x0 = [1.5;0.6]`. Hand in suitable and informative plots and comment your observations.

Hints:

1. For the computation of $H^{\pm\frac{1}{2}}$ consider the eigenvalue and eigenvector decomposition via Matlab function `eig`.
2. Remember to compute and use the exact step size t in the algorithm.

Exercise 17 (Cauchy-step property)
The Cauchy step is defined as $s_a^c = -t_a \nabla f(x_a)$, where t_a is giving by (see the lecture notes)

$$t_a = \begin{cases} \frac{\Delta_a}{\|\nabla f(x_a)\|} & \text{if } \nabla f(x_a)^\top H_a \nabla f(x_a) \le 0, \\ \min\left(\frac{\Delta_a}{\|\nabla f(x_a)\|}, \frac{\|\nabla f(x_a)\|^2}{\nabla f(x_a)^\top H_a \nabla f(x_a)}\right) & \text{if } \nabla f(x_a)^\top H_a \nabla f(x_a) > 0. \end{cases}$$

Once the Cauchy point $x_a^c = x_a + s_a^c$ is computed, show that there is a sufficient decreasing in the quadratic model, i.e, the Cauchy step satisfies

$$\psi_a(0) - \psi_a(t_a) \ge \frac{1}{2}\|\nabla f(x_a)\| \min\left(\Delta_a, \frac{\|\nabla f(x_a)\|}{1 + \|H_a\|}\right).$$

Exercise 18 (Classical Newton method) (4 Points)
Consider the functions $f, g : \mathbb{R} \to \mathbb{R}$, given by

$$f(x) = x^3 - 2x + 2, \quad \text{and} \quad g(x) = \sin(x).$$

1. Show that for the starting point $x_0 = 0$, the classical Newton iteration of f has two accumulation points which are both not roots of f. Find another initial point which does not lead to a convergence of the Newton method applied on f.

2. Find a starting point x_0 such that the Newton iteration for g tends to $+\infty$.

3. Explain why the methods do not converge to a root of the functions by a suitable plot.