Universität Konstanz Sommersemester 2014
Fachbereich Mathematik und Statistik
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Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 2

Deadline for hand-in: 2014/05/21 at lecture

Exercise 4 (4 Points)
Consider the quadratic function $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle + \gamma$$

where $Q \in \mathbb{R}^{n\times n}$ is symmetric, $c \in \mathbb{R}^n$ and $\gamma \in \mathbb{R}$, where $\langle \cdot\,, \cdot\rangle$ denotes the Euclidean inner product in $\mathbb{R}^n$.

Show directly, i.e. without using any theorem from the scriptum, that the following holds:

$$f \text{ is convex} \Leftrightarrow Q \text{ is positive semidefinite.}$$

Exercise 5
Consider the quadratic function $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle + \gamma$$

where $Q \in \mathbb{R}^{n\times n}$ symmetric and positive definite, $c \in \mathbb{R}^n$ and $\gamma \in \mathbb{R}$. Let $x^k \in \mathbb{R}^n$ be arbitrary and $d^k \in \mathbb{R}^n$ a descent direction of f in x^k for a $k \in \mathbb{N}$.

Find the (exact) step size s^* in direction d^k such that the decreasing of $f(x^k + s^* d^k)$ is maximal.

Exercise 6
Consider the general descent method (as known from the lecture) for the function

$$f : \mathbb{R} \to \mathbb{R}, \qquad f(x) = x^2,$$

with starting point $x^0 := 1$ and the following directions $d^k \in \mathbb{R}$ and step-sizes $t^k \in \mathbb{R}$:

1. $d^k := -1$, $t^k := \left(\frac{1}{2}\right)^{k+2}$ with $k \in \mathbb{N}_0$,
2. $d^k := (-1)^{k+1}$, $t^k := 1 + \frac{3}{2^{k+2}}$ with $k \in \mathbb{N}_0$.

Verify that these choices lead to a decrease of the function f. For that, present the sequence x^k generated by the steepest descent method using induction with respect to k. Further determine in each case the limit $\lim_{k\to\infty} f(x^k)$ and compare it to the minimum of $f(x)$: what can you observe?

Exercise 7

You organized a party in your living room. You started a conversation with a friend, who is talking to you from position B (see Figure 1) while you are standing in position $x_0 = 1$. Suddenly you realize that a person in position A is talking to someone close to her/him

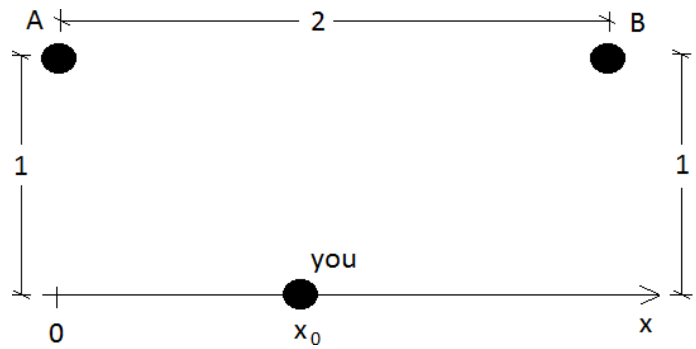

Figure 1: Positions and the party.

about something that really interests you: you'd like to hear what they are saying. But you are very polite and you don't want to tell your friend to stop talking so you decide to move, along the x-axis, toward the person of interest, for understanding what he/she is saying. Assume that your friend and the person of interest are talking using the same volume and that the power of the voice you can hear is the reciprocal of the square of the distance, find the optimal position x^* you should be in, such that you maximize the so-called "person of interest-to-talkative friend" ratio, defined as[1]

$$g(x) = \frac{\text{power of the voice of the person of interest}}{\text{power of the voice of the interfering friend}}.$$

[1]Hints: You can equivalently reformulate the ratio using, in an appropriate way, the distances, which are functions of the minimization variable x. You might formulate the problem as an equivalent minimization problem.