

Optimierung

<http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/>

Sheet 3

Deadline for hand-in: 2014/06/04 at lecture

Exercise 8

Let $b \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$.

Show that $x_0 \in \mathbb{R}^n$ is a minimal point of $\varphi : \mathbb{R}^n \rightarrow \mathbb{R}$, $x \mapsto \varphi(x) := \|Ax - b\|_2$ if and only if the *Gaussian normal equation* $A^\top Ax_0 = A^\top b$ holds.

Exercise 9

Use the characterization given in Exercise 8 to solve the following linear regression problem:

Find a vector of parameters $x = (x_1, x_2) \in \mathbb{R}^2$ such that the corresponding regression line $\gamma_x : \mathbb{R} \rightarrow \mathbb{R}$, defined by $\gamma_x(t) := x_1 + tx_2$, approximates the following measuring points

t_i	1975	1980	1985	1990	1995
γ_i	30	35	38	42	44

optimally, i.e. such that $x^* = (x_1^*, x_2^*)$ solves the optimization problem:

$$x^* = \operatorname{argmin}_{x \in \mathbb{R}^2} \sum_{i=1}^5 (\gamma_i - \gamma_x(t_i))^2.$$

Exercise 10

(4 Points)

Let a, b, c, d vectors in \mathbb{R}^n , with b, d linear independent. Consider the parametrization of two straight lines $x(s) : \mathbb{R} \rightarrow \mathbb{R}^n$ and $y(t) : \mathbb{R} \rightarrow \mathbb{R}^n$ with

$$x(s) := a + sb, \quad y(t) := c + td \quad (s, t \in \mathbb{R}).$$

Find the critical points of the distance function

$$(s, t) \mapsto \|x(s) - y(t)\|$$

where $\|\cdot\|$ is intended as Euclidean norm and characterize them.

Exercise 11

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ convex function and $a, b \in \mathbb{R}$, with $a < b$ and $x \in [a, b]$. Show that

$$\frac{f(x) - f(a)}{x - a} \leq \frac{f(b) - f(a)}{b - a} \leq \frac{f(b) - f(x)}{b - x}$$

showing first the following result

$$f(x) \leq \frac{x - a}{b - a} f(b) + \frac{b - x}{b - a} f(a).$$