Universität Konstanz Sommersemester 2014
Fachbereich Mathematik und Statistik
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Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Program 2

Submission by E-Mail: 2014/06/18, 10:00 h

Implementation of the Projected Gradient Method.

Part 1: Generate a file `projection.m` and implement the function

```
function [px] = projection(x, a, b)
```

with the current point `x` $\in \mathbb{R}^n$, lower bound `a` $\in \mathbb{R}^n$ and upper bound `b` $\in \mathbb{R}^n$ as input arguments. The function returns the (pointwise) projected point `px` $\in \mathbb{R}^n$ according to the projection

$$P : \mathbb{R}^n \to \Omega := \{x \in \mathbb{R}^n \mid \forall i = 1, ..., n : \ a_i \leq x_i \leq b_i\}.$$

Test your function for the rectangular 2-D domain defined by the lower bound (lower left corner) $a = (-1; -1)$ and the upper bound (upper right corner) $b = (1; 1)$: compute the projection $P(x)$ of points $x = y + td \in \mathbb{R}^2$ with $y \in \mathbb{R}^2$ as given in the table below, direction $d = (1.5; 1.5) \in \mathbb{R}^2$, step sizes $t = 0$ and $t = 1$. For validation compare your results to the projections given in the table:

Points y:	$P(x)$ for $t = 0$	$P(x)$ for $t = 1$
$(-2; -2)$	$(-1; -1)$	$(-0.5; -0.5)$
$(-1; -1)$	$(-1; -1)$	$(0.5; 0.5)$
$(-0.5; 0.5)$	$(-0.5; 0.5)$	$(1; 1)$
$(2; 0.5)$	$(1; 0.5)$	$(1; 1)$
$(1; -0.5)$	$(1; -0.5)$	$(1; 1)$

Table 1: Testing points and their projections with respect to t

Part 2: Write a function

```
function [t] = modarmijo(fhandle, x, d, t0, alpha, beta, a, b)
```

for the Armijo step size strategy with the modified termination condition

$$f(x(t, d)) - f(x) \leq -\frac{\alpha}{t}||x - x(t, d)||^2$$

with current point `x`, descent direction `d`, initial step size `t0`, `alpha` and `beta` for the Armijo rule and `a` and `b` for the projection rule and $x(t, d)$ defined as

$$x(t, d) := P(x - td).$$

Part 3: Implement the gradient projection algorithm with direction $d_k := -\frac{\nabla f(x_k)}{\|\nabla f(x_k)\|}$. Generate a file `gradproj.m` for the function

```
function [X] = gradproj(fhandle, x0, epsilon, nmax, t0, alpha, beta, a, b)
```

with initial point `x0`, parameter `epsilon` for the termination condition $\|\nabla f(x_k)\| < \epsilon$ and the additional termination condition $\|x - x(1)\| < \epsilon$, `nmax` for the maximal number of iteration steps, parameters `t0`, `alpha` and `beta` for the Armijo rule and `a` and `b` for the projection rule. Modify therefore the gradient method `gradmethod.m`.

The program should return a matrix `X = [x0; x1; x2; ...]` containing the whole iterations.

Part 4: Call the function `gradproj` from a main file `main.m` to test your program for the Rosenbrock function

```
function [f,g] = rosenbrock(x)
```

with input argument `x` $\in \mathbb{R}^2$ and output arguments the corresponding function value `f` $\in \mathbb{R}$ and gradient `g` $\in \mathbb{R}^2$. Use the following parameters: visualize the results in suitable plots and write your observations in the written report:

`epsilon=1.0e-2`, `N=1.5e+3`, `t0=1`, `alpha=1.0e-2`, `beta=0.5` and

1. `x0=[1;-0.5]`, `a=[-1;-1]` and `b=[2;2]`
2. `x0=[-1;-0.5]`, `a=[-2;-2]` and `b=[2;0]`
3. `x0=[-2;2]`, `a=[-2;-2]` and `b=[2;2]`