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Numerische Verfahren der restringierten Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 1

Deadline for hand-in: 30.10.2014 at lecture

Exercise 1

(2 Points)

Consider the problem of finding the point on the parabola $y = \frac{1}{5}(x-1)^2$ that is close to (x,y) = (1,2), in the Euclidean norm sense. We can formulate this as

min $f(x, y) = (x - 1)^2 + (y - 2)^2$ u.d.N. $(x - 1)^2 = 5y$.

a) Find all the KKT points for this problem. Are all points regular points?

b) Which of these points are solutions?

Exercise 2 Solve the problem

$$\min_{x} x_1 + x_2 \quad \text{s.t.} \ x_1^2 + x_2^2 = 1$$

by eliminating the variable x_2 . Show that the choice of sign for the square root operation during the elimination process is critical; the "wrong" choice leads to an incorrect answer.

Exercise 3

Consider the problem

$$\min_{x} \left(x_1 - \frac{3}{2} \right)^2 + (x_2 - t)^4 \quad \text{s.t.} \quad \begin{bmatrix} 1 - x_1 - x_2 \\ 1 - x_1 + x_2 \\ 1 + x_1 - x_2 \\ 1 + x_1 + x_2 \end{bmatrix} \ge 0,$$

where t is a parameter to be fixed prior to solving the problem.

- a) For what values of t does the point $x^* = (1, 0)^{\top}$ satisfy the KKT conditions?
- b) Show that when t = 1, only the first constraint is active at the solution, and find the solution.