Universität Konstanz
Fachbereich Mathematik und Statistik
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Wintersemester 14/15

Numerische Verfahren der restringierten Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 4

Deadline for hand-in: 11.12.2014 at lecture

Exercise 10 (2 Points)

Let A be the matrix in the equality constraint given by $[B|N]$, with B invertible. Consider the matrices Y and Z given by

$$Y = \begin{bmatrix} B^{-1} \\ 0 \end{bmatrix} \quad \text{and} \quad Z = \begin{bmatrix} -B^{-1}N \\ I \end{bmatrix}.$$

Show that their columns are linearly independent and that the assumptions for the *null space method* are satisfied. Furthermore, let $x = [x_B \; x_N]^\top$. Write the optimization problem

$$\begin{aligned} \min \; & \sin(x_3 + x_4) + x_1^2 + \tfrac{1}{3}(x_5 + x_6^4 + x_2/2) \\ \text{subject to} \quad & x_1 + 8x_3 - 6x_4 + 9x_5 + 4x_6 = 6 \\ & 4x_2 + 3x_3 + 2x_4 - x_5 + 6x_6 = -4 \end{aligned}$$

in reduced form by using the matrices Y and Z. Show first that $x_B = B^{-1}b - B^{-1}Nx_n$ is satisfied.

Exercise 11

Assuming that the conditions of Lemma 3.1 (see lecture notes) are satisfied, compute the inverse of the KKT-Matrix (3.1).

Exercise 12

The problem of finding the shortest Euclidean distance from a point x_0 to the hyperplane $\{x \mid Ax = b\}$, where A has full row rank, can be formulated as a quadratic program. Write the problem in the form ($\mathbf{QP}_{Gl}$), derive the KKT-system (3.2) and determine the solutions x^* and λ^* explicitly. Further, show that in the special case in which A is a row vector, the shortest distance from x_0 to the solution set of $Ax = b$ is $|b - Ax_0|/\|A\|_2$.