Universität Konstanz
Fachbereich Mathematik und Statistik
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Wintersemester 14/15

Numerische Verfahren der restringierten Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Program 1 (6 Points)

Submission by E-Mail: 14.11.2014, 18:00 h

Note:

- Work in **groups of 2 to 3 members**!
- Do not forget to write **name** and **email adress** of the authors in each file and document your code well!
- Only **running programs** will be considered!
- Stick to the **given function and parameter definitions** as described below! You should not modify them in name or concerning the input and output arguments.

Consider the domain $\Omega = (0, 10)$ and the following Poisson problem

$$\begin{cases} \Delta \mathrm{y}(x) = \mathrm{b}(x) & \text{in } \Omega \\ \mathrm{y}(0) = \mathrm{g}(0) \\ \mathrm{y}(10) = \mathrm{g}(10). \end{cases}$$

Once one discretizes the domain Ω with n points, using the stepsize $h = 1/(n+1)$, the problem can be numerically solved by solving, only for the inner points $Ay = b$, with b the discretization of b, $b, y \in \mathbb{R}^n$ and A

$$A = \frac{1}{h^2} \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix} \in \mathbb{R}^{n \times n}$$

the resulting matrix of the discretization of the Laplace operator with central differences. Let us now consider the following optimization problem

$$\min J(x) = \frac{1}{2} y^\top Q y + y^\top d \quad \text{s.t} \quad Ay = b,$$

with $Q \in \mathbb{R}^{n \times n}$, $d \in \mathbb{R}^n$. Implement the following Matlab function

```
[y,lambda]=myquadprog(Q,d,A,b,flag)
```

for solving the quadratic program via direct solve of the system, where `y` and `lambda` are column vectors and `flag`$\in$`{1,2,3}` should set the system solver of the function, according to the following association: `1` for the QR, `2` for LU and `3` (default) backslash (use the appropriate Matlab functions for the matrix decomposition).
Implementing a `mymain` file which will define all the necessary matrices, calls `myquadprog` and plots the results, using the following setting:

$$\mathrm{b}(x) = 2\frac{\cos x}{e^x}, \quad \mathrm{g}(x) = \frac{\sin x}{e^x}$$

and d the vector with the evaluations of the function

$$\mathrm{d}(x) = \frac{\sin 0.2}{e^{0.2}} + (x - 0.2)\left(e^{-x}\cos x - e^{-x}\sin x\right)$$

in the discretized domain Ω. Test your program with $Q = I \in \mathbb{R}^n$ and solve the quadratic program using the three solvers for $n \in \{1 \cdot 10^2, 500, 1 \cdot 10^3\}$.
Put in your written report the plots of the solutions and your observations.