Universität Konstanz
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Wintersemester 14/15

Numerische Verfahren der restringierten Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Program 2 (6 Points)

Submission by E-Mail: 12.12.2014, 12:00 h

Program the Predictor-Corrector Algorithm (Mehrotra, Algorithm 2.16) in MATLAB. Test your code on the linear programming problem

$$\min c^\top x, \quad \text{subject to } Ax = b,\ x \geq 0$$

and corresponding dual problem

$$\max b^\top \lambda, \quad \text{subject to } A^\top \lambda + \mu = c,\ \mu \geq 0$$

generated by choosing the matrix $A \in \mathbb{R}^{m \times n}$ randomly ($m < n$), and then setting $x, c, \mu \in \mathbb{R}^n$ and $b, \lambda \in R^m$ as follows:

$$\begin{aligned}
x_i &= \begin{cases} \text{random positive number} & i = 1, 2, \ldots, m, \\ 0 & i = m+1, m+2, \ldots, n, \end{cases} \\
\mu_i &= \begin{cases} \text{random positive number} & i = m+1, m+2, \ldots, n, \\ 0 & i = 1, 2, \ldots, m, \end{cases} \\
\lambda &= \text{random vector}, \\
c &= A^\top \lambda + \mu, \\
b &= Ax.
\end{aligned}$$

Choose the starting point (x^0, λ^0, μ^0) with the components of x^0 and μ^0 set to large positive values.

Compare the computed numerical solution obtained by the algorithm with the exact solution used to generate the examples to verify the correctness of your code. Try different values for m, n and the tolerance ε (`tol`), for example $n = 30, m = 20$ and `tol`$= 10^{-12}$.

Use the provided template `mylinprog.m` as a guideline for the implementation of your algorithm. Check dimensions of input arguments and give corresponding error messages. Stop the algorithm if `maxiter` is reached and provide the user with a corresponding note. Additionally submit a script file called `mymain.m` to run the above generated example.

Listing 1: Template for the MATLAB routine mylinprog.m

```
function [x,lambda,mu] = mylinprog(c,A,b,tol,maxiter, ...
                                                    x0,lambda0,mu0)
%MYLINPROG Linear programming
%  [x,lambda,mu] = MYLINPROG(c,A,b,tol,maxiter,x0,lambda0,mu0)
%  attempts to solve the linear programming problem utilizing
%  the predictor corrector algorithm (Mehrotra):
%
%          min c'*x    subject to    A*x = b, x >= 0
%
%  Input : c ......... vector of dimension nx1
%          A ......... matrix of dimension mxn
%          b ......... vector of dimension mx1
%          tol ....... tolerance for the stopping criteria
%          maxiter ... maximum number of allowed iterations
%          x0 ........ initial guess for the solution x
%                      (dimension nx1)
%          lambda0 ... initial guess for the Lagrange
%                      multiplier lambda (dimension mx1)
%          mu0 ....... initial guess for the Lagrange
%                      multiplier mu (dimension nx1)
%
%  Output: x ......... numerical solution for x
%          lambda .... numerical solution for Lagrange
%                      multiplier lambda
%          mu ........ numerical solution for Lagrange
%                      multiplier mu
%
```