

Optimierung

<http://www.math.uni-konstanz.de/numerik/personen/rogg/de/teaching/>

Sheet 1

Tutorial in calender week 18

Exercise 1

Determine and identify the local critical point(s) of the Rosenbrock function

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}, \quad f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

Exercise 2

Show that the function

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}, \quad f(x_1, x_2) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$$

has only one stationary point, and that it is neither a maximum nor minimum, but a saddle point. Sketch the contour lines of f (you can also use Matlab).

Exercise 3

Consider the function

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}, \quad f(x_1, x_2) = 3x_1^4 - 4x_1^2x_2 + x_2^2.$$

Prove that $\tilde{x} = (0, 0)$ is a critical point of f . Show further, that a restriction of f on any line¹ through \tilde{x} , has a strict local minimum in \tilde{x} . Is \tilde{x} a local minimizer of f ?

¹The restriction of f on the line γ is defined as $g(t) = f(\gamma(t))$, $t \in \mathbb{R}$, with $\gamma(t) := \tilde{x} + td$, where $d \in \mathbb{R}^2 \setminus \{0\}$ is an arbitrary but fixed direction.