Universität Konstanz
Fachbereich Mathematik und Statistik
Prof. Dr. Stefan Volkwein
Sabrina Rogg

Sommersemester 2015

Optimierung

http://www.math.uni-konstanz.de/numerik/personen/rogg/de/teaching/

Sheet 2

Tutorial: 12th May

Exercise 4
Consider the quadratic function $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle + \gamma$$

where $Q \in \mathbb{R}^{n\times n}$ is symmetric, $c \in \mathbb{R}^n$, $\gamma \in \mathbb{R}$ and where $\langle \cdot\,, \cdot\rangle$ denotes the Euclidean inner product in $\mathbb{R}^n$.

Show directly, i.e. without using any theorem from the scriptum, that the following holds:

$$f \text{ is convex} \Leftrightarrow Q \text{ is positive semidefinite.}$$

Exercise 5
Consider the quadratic function $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle + \gamma$$

where $Q \in \mathbb{R}^{n\times n}$ is symmetric and positive definite, $c \in \mathbb{R}^n$ and $\gamma \in \mathbb{R}$. Let $x^k \in \mathbb{R}^n$ be arbitrary and $d^k \in \mathbb{R}^n$ an arbitrary descent direction of f in x^k.

Find the (exact) step size s^* in direction d^k such that the decreasing of $f(x^k + s^* d^k)$ is maximal.

Exercise 6
Consider the unconstrained optimization problem

$$\min_{x\in\mathbb{R}^2} f(x) := -e^{-((x_1-\pi)^2+(x_2-\pi)^2)}, \tag{1}$$

with $f : \mathbb{R}^2 \to \mathbb{R}$. Show that f has only one stationary point and that $x^* = (\pi, \pi)^\top$ is the global solution to problem (1). We modify the objective functional as follows:

$$\tilde{f}(x) = f(x) + \alpha \sin(x_1)\cos(x_2 + \frac{\pi}{2}),$$

with $\alpha = 0.1$. Visualize the function $\tilde{f}$ using Matlab. What do you observe concerning local and global minima?