Universität Konstanz
Fachbereich Mathematik und Statistik
Prof. Dr. Stefan Volkwein
Sabrina Rogg

Sommersemester 2015

# Optimierung

http://www.math.uni-konstanz.de/numerik/personen/rogg/de/teaching/

## Sheet 5

### Tutorial: 23 June

**Exercise 14** (Classical Newton method)
Consider the functions $f, g : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = x^3 - 2x + 2 \quad \text{and} \quad g(x) = \sin(x).$$

(1) Show that for the starting point $x_0 = 0$, the classical Newton iteration of $f$ has two accumulation points which are both not roots of $f$.

(2) Find a starting point $x_0$ such that the Newton iteration is of the form $x_k = x_0 + k\pi$, $k \in \mathbb{N}_{>0}$.

(3) Generate suitable plots (Matlab) for (1) and (2).

**Exercise 15** (Damped Newton)
Consider to use the classical Newton method for finding the root of $g(x) = \arctan(x)$. Let $x_k = 5$. Compute the next iteration point $x_{k+1}$ according to the Newton algorithm

$$\begin{aligned} g'(x_k)\Delta x_k &= -g(x_k); \\ x_{k+1} &= x_k + \Delta x_k. \end{aligned}$$

The damped Newton algorithm is defined as

$$\begin{aligned} g'(x_k)\Delta x_k &= -g(x_k); \\ x_{k+1} &= x_k + \alpha_k \Delta x_k, \end{aligned}$$

with $\alpha_k \in (0, 1)$ computed by a line search procedure such that

$$|g(x_k + \alpha_k \Delta x_k)| \leq (1 - \delta\alpha_k)|g(x_k)|, \quad \delta \in (0, 1).$$

Let $\delta = 0.5$. We start from $\alpha = 1$ in the line search procedure and divide it by 2 every time the condition is not satisfied. Compute the point $x_{k+1}$ using the damped Newton algorithm and compare it to the one returned by the classical Newton method. What do you conclude?

**Exercise 16**
Let $f \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R})$ be the quadratic function

$$f(x) = \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle + \gamma,$$

with $Q \in \mathbb{R}^{n\times n}$ symmetric and positive definite, $c \in \mathbb{R}^n$ and $\gamma \in \mathbb{R}$. Let $x^0 \in \mathbb{R}^n$ and $H$ be a symmetric positive definite matrix.

Define $\tilde{f}(x) := f(H^{-\frac{1}{2}}x)$ and $\tilde{x}^0 := H^{\frac{1}{2}}x^0$. Let $(\tilde{x}^k)_{k\in\mathbb{N}}$ be a sequence generated by the steepest descent method,

$$\tilde{x}^{k+1} = \tilde{x}_k + \tilde{t}_k\tilde{d}^k \quad \text{with} \quad \tilde{d}^k = -\nabla\tilde{f}(\tilde{x}_k)$$

and $\tilde{t}_k = t(\tilde{d}^k)$ the optimal stepsize choice as determined in Exercise 5 (for $\tilde{f}$).

Let $(x^k)_{k\in\mathbb{N}}$ be generated by the gradient-like method with preconditioner $H$,

$$x^{k+1} = x^k + t_k d^k \quad \text{with} \quad d^k = H^{-1}(-\nabla f(x^k))$$

and $t_k = t(d^k)$ the optimal stepsize choice as determined in Exercise 5.

Show (by induction) that the two optimization methods are equivalent, i.e., for all $k \in \mathbb{N}$ it holds:

$$x^k = H^{-\frac{1}{2}}\tilde{x}^k.$$