Universität Konstanz
Fachbereich Mathematik und Statistik
Prof. Dr. Stefan Volkwein
Sabrina Rogg

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Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 2

Deadline for hand-in: 2.05.2016 at lecture

Exercise 4 (2 Points)

Consider the quadratic function $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle + \gamma$$

where $Q \in \mathbb{R}^{n\times n}$ is symmetric and positive definite, $c \in \mathbb{R}^n$ and $\gamma \in \mathbb{R}$. Let $x^k \in \mathbb{R}^n$ be arbitrary and $d^k \in \mathbb{R}^n$ an arbitrary descent direction of f in x^k.

Find the (exact) step size s^* in direction d^k, i.e.

$$s^* = \operatorname*{argmin}_{s>0} f(x^k + sd^k).$$

Exercise 5 (3 Points)

1. Let $X \subset \mathbb{R}^n$ convex and $f : X \to \mathbb{R}$ convex with $f(X) \subset I$ for some open interval $I \subset \mathbb{R}$. Let $g : I \to \mathbb{R}$ be a convex and monotone increasing function.
   a) Show that $h : X \to \mathbb{R}$ defined by $h := g \circ f$ is convex.
   b) Is a) still true when the function g is not monotone increasing?
2. Is the product of two convex functions convex?

Exercise 6 (3 Points)

Consider the unconstrained optimization problem

$$\min_{x\in\mathbb{R}^2} f(x) := -e^{-((x_1-\pi)^2+(x_2-\pi)^2)}, \qquad (1)$$

with $f : \mathbb{R}^2 \to \mathbb{R}$. Show that f has only one stationary point and that $x^* = (\pi,\pi)^\top$ is the global solution to problem (1). We modify the objective functional as follows:

$$\tilde{f}(x) = f(x) + c\sin(x_1)\cos(x_2 + \frac{\pi}{2}),$$

with $c = 0.1$. Visualize the functions f and $\tilde{f}$ using Matlab. What do you observe concerning local and global minima?