Universität Konstanz
Fachbereich Mathematik und Statistik
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Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 3

Deadline for hand-in: 17.05.2016, 10:00 h, G413

Exercise 7 (3 Points)

Let $b \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$.

Show that $x^* \in \mathbb{R}^n$ is a minimal point of $\varphi : \mathbb{R}^n \to \mathbb{R}$, $x \mapsto \varphi(x) := ||Ax - b||_2^2$ if and only if the *Gaussian normal equation* $A^\top A x^* = A^\top b$ holds.

Exercise 8 (2 Points)

For a fixed vector $x = (x_1, x_2) \in \mathbb{R}^2$ the corresponding regression line $\gamma_x : \mathbb{R} \to \mathbb{R}$ is defined by $\gamma_x(t) := x_1 + t x_2$. Use the characterization given in Exercise 7 to solve the following linear regression problem:

Find a vector $x^* = (x_1^*, x_2^*) \in \mathbb{R}^2$ such that γ_{x^*} approximates the following measuring points

t_i	1975	1980	1985	1990	1995
γ_i	30	35	38	42	44

optimally, i.e. such that x^* solves the optimization problem:

$$x^* = \underset{x \in \mathbb{R}^2}{\operatorname{argmin}} \sum_{i=1}^{5} (\gamma_i - \gamma_x(t_i))^2.$$

Exercise 9 (3 Points)

Let a, b, c, d vectors in $\mathbb{R}^n$, with b, d linear independent. Consider the parametrization of two straight lines $x(s) : \mathbb{R} \to \mathbb{R}^n$ and $y(t) : \mathbb{R} \to \mathbb{R}^n$ with

$$x(s) := a + sb \ , \qquad y(t) := c + td \qquad (s, t \in \mathbb{R}).$$

Determine the global minimal point of the squared distance function

$$(s, t) \mapsto ||x(s) - y(t)||_2^2.$$