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Fachbereich Mathematik und Statistik
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Sommersemester 2016

Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Program 2 (6 Points)

Submission by E-Mail: 30.05.2016, 10:00 h

Local Newton method

Implement the local Newton method for optimization known from the lecture (Algorithm 5.6 with $F = \nabla f$). Write a function

```
function [X] = newtonmethod(fhandle, x0, epsilon, nmax)
```

with input arguments

- `fhandle`: function handle to a function of form `[f,g,H] = functionname(x)` (the output values are the function value, the gradient and the Hessian matrix corresponding to the input argument `x`).
- `x0`: initial point
- `epsilon`: tolerance for the termination condition $\|\nabla f(x^k)\| < \epsilon$
- `nmax` : maximum number of iterations

The program should return a matrix `X = [x0, x1, x2, ...]` containing the whole iterations.

Numerical experiments:
Write a main file `main.m` for the tests below.

1. Implement a function file `fun1.m` of the above form for the function

$$f_1(\boldsymbol{x}) = (x_1 - 2)^4 + x_2^2(x_1 - 2)^2 + (x_2 + 1)^2.$$

   Write down the gradient and Hessian of f_1 in your report. What is the global minimizer of f_1? Apply the functions `newtonmethod` and `gradmethod` (from Pogram 1) for optimization. Use `x0=[1;1]`, `epsilon = 1e-6`, `nmax = 50` and `t0 = 1`, `amax = 30`, `alpha = 1e-2`, `beta = 0.5`. Visualize the iterates of both methods along with the contour lines of the objective function (in one plot). In addition, plot the values of the objective function in semilog scale. What can you say about convergence?

2. Implement a function file `fun2.m` for the function

$$f_2(\boldsymbol{x}) = (x_1 - 2)^4 + (x_1 - 2x_2)^2.$$

Again, name the gradient and Hessian of this function in your report. What is the global minimizer of f_2? Apply the function `newtonmethod` with `x0=[0;3]`, `epsilon = 1e-6` and `nmax = 50`. Visualize the iterates along with the contour lines of the objective function. On what line do the iterates except the initial point lie? Hint: Hessian.
Look at the convergence behaviour. Which property is not given here compared to point 1? How many iterations does the the function `gradmethod` with `t0 = 1`, `amax = 30`, `alpha = 1e-2` and `beta = 0.5` need to reach the termination criterion?

3. Write a function file `negativeCosine.m` for the negative cosine function. Use the parameters `epsilon = 1e-5` and `nmax = 50`. As initial points choose `x0 = 1.1655`, `1.1656, 1.9, atan(-pi)`. Explain the results you get and use suitable plots for showing `X`. For comparison plot also the iterates you obtain by applying the function `gradmethod` with `t0 = 1`, `alpha = 1e-2`, `beta = 0.5` and `amax = 30`.