Universität Konstanz
Fachbereich Mathematik und Statistik
Prof. Dr. Stefan Volkwein
Jianjie Lu, Sabrina Rogg

Wintersemester 16/17

Numerische Verfahren der restringierten Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 1

Deadline for hand-in: Will be named.

Exercise 1 (2 points)

Consider the problem of finding the point on the parabola $y = \frac{1}{5}(x-1)^2$ that is close to $(x, y) = (1, 2)$, in the Euclidean norm sense. We can formulate this as

$$\min f(x,y) = (x-1)^2 + (y-2)^2 \quad \text{u.d.N. } (x-1)^2 = 5y.$$

a) Find all the KKT points for this problem. Are all points regular points?

b) Which of these points are solutions?

Exercise 2

Solve the problem

$$\min_x x_1 + x_2 \quad \text{s.t. } x_1^2 + x_2^2 = 1$$

by eliminating the variable x_2. Show that the choice of sign for the square root operation during the elimination process is critical; the "wrong" choice leads to an incorrect answer.

Exercise 3

Consider the problem

$$\min_x \left(x_1 - \frac{3}{2}\right)^2 + (x_2 - t)^4 \quad \text{s.t. } \begin{bmatrix} 1 - x_1 - x_2 \\ 1 - x_1 + x_2 \\ 1 + x_1 - x_2 \\ 1 + x_1 + x_2 \end{bmatrix} \geq 0,$$

where t is a parameter to be fixed prior to solving the problem.

a) For what values of t does the point $x^* = (1, 0)^\top$ satisfy the KKT conditions?

b) Show that when $t = 1$, only the first constraint is active at the solution, and find the solution.