Universität Konstanz
Fachbereich Mathematik und Statistik
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Wintersemester 16/17

Numerische Verfahren der restringierten Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 4

Deadline for hand-in: 10.01.2017 at lecture

Exercise 9 (2 Points)

Let $A = [B|N]$ be the matrix in the equality constraint $Ax = b$, with $B \in \mathbb{R}^{m\times m}$ invertible, $N \in \mathbb{R}^{m\times n-m}$ and $x = [x_B\ x_N]^\top$. Consider the matrices Y and Z given by

$$Y = \begin{bmatrix} B^{-1} \\ 0 \end{bmatrix} \quad \text{and} \quad Z = \begin{bmatrix} -B^{-1}N \\ I \end{bmatrix},$$

Show that their columns are linearly independent and that the constraint implies

$$x_B = B^{-1}b - B^{-1}Nx_N.$$

Write the optimization problem

$$\begin{aligned} &\min \sin(x_3 + x_4) + x_1^2 + \frac{1}{3}(x_5 + x_6^4 + x_2/2) \\ &\text{subject to} \quad x_1 + 8x_3 - 6x_4 + 9x_5 + 4x_6 = 6 \\ &\qquad\qquad\quad 4x_2 + 3x_3 + 2x_4 - x_5 + 6x_6 = -4 \end{aligned} \tag{1}$$

in the above form by defining the matrices B and N. Then, write (1) as a problem depending only on x_N.

Exercise 10

Assuming that the conditions of Lemma 3.1 (see lecture notes) are satisfied, compute the inverse of the KKT-Matrix (3.1).

Exercise 11

The problem of finding the shortest Euclidean distance from a point x_0 to the hyperplane $\{x \mid Ax = b\}$, where A has full row rank, can be formulated as a quadratic program. Write the problem in the form $(\mathbf{QP}_{Gl})$, derive the KKT-system (3.2) and determine the solutions x^* and λ^* explicitly. Further, show that in the special case in which A is a row vector, the shortest distance from x_0 to the solution set of $Ax = b$ is $|b - Ax_0|/\|A\|_2$.