Universität Konstanz
Fachbereich Mathematik und Statistik
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Wintersemester 16/17

Numerische Verfahren der restringierten Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 6

Deadline for hand-in: 07.02.2017 at lecture

Exercise 15 (2 Points)

Given the problem

$$\min_{x\in\mathbb{R}^2} f(x) := -x_1 - x_2 \quad \text{s.t.} \quad g(x) := -x \le 0,\, e(x) := x_1^2 + x_2^2 - 1 = 0. \tag{1}$$

a) Sketch the admissible set and the cost function (use contour lines for the cost function).

b) Calculate the solution of (1) and the corresponding Lagrange multipliers.

c) Let $x^k = (-1/2, -1/2)^\top$ be given. Sketch the constraints of the SQP subproblem and show that the corresponding admissible set is empty.

Exercise 16 (2 Points)

Given the problem

$$\min_{x\in\mathbb{R}^n} f(x) \quad \text{subject to} \quad e(x) = 0, \tag{2}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ and $e : \mathbb{R}^n \to \mathbb{R}^m$ are $\mathcal{C}^2$ functions. The *augmented Lagrange function* for (2) is defined as

$$L_\alpha(x, \lambda) := f(x) + \lambda^\top e(x) + \frac{\alpha}{2}\|e(x)\|^2$$

with $\alpha \ge 0$.

a) Show that all KKT pairs (x, λ) satisfy

$$\nabla L_\alpha(x, \lambda) = 0.$$

b) Let (x^*, λ^*) be a KKT pair that satisfies the second order sufficient optimality condition. Show that the Hessian matrix $\nabla_{xx} L_\alpha(x^*, \lambda^*)$ is positive definite for sufficiently large α. Hence, x^* is a *global* minimum of $L_\alpha(\cdot, \lambda^*)$ provided that α is sufficiently large.

Exercise 17 (2 Points)

Given the problem

$$\min -x_1 x_2^2 \quad \text{subject to} \quad x_1^2 + x_2^2 = 1. \tag{3}$$

Show that $x^* = \left(\sqrt{\frac{1}{3}}, \pm\sqrt{\frac{2}{3}}\right)^\top$ are the solutions of (3), with Lagrange multiplier $\lambda^* = \sqrt{\frac{1}{3}}$. We consider the Hessian matrix $\nabla_{xx} L_\alpha(x^*, \lambda^*)$ of the augmented Lagrange function from Exercise 16. Visualize (Matlab) the contour lines of $L_\alpha(x, \lambda^*)$ for different values α (e.g. $\alpha = 0, 0.6$). For which values of α is $\nabla_{xx} L_\alpha(x^*, \lambda^*)$ positive definite?