Universität Konstanz
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Numerische Verfahren der restringierten Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Program 2 (6 Points)

Submission by E-Mail to jianjie.lu@uni-konstanz.de:
10.01.2017, 10:00 h

Program the Predictor-Corrector Algorithm (Mehrotra, Algorithm 2.16). Test your code on the linear programming problem

$$\min c^\top x, \quad \text{subject to } Ax = b,\ x \geq 0$$

with corresponding dual problem

$$\max b^\top \lambda, \quad \text{subject to } A^\top \lambda + \mu = c,\ \mu \geq 0$$

generated by choosing the matrix $A \in \mathbb{R}^{m\times n}$ randomly ($m < n$), and then setting $x^*, c, \mu^* \in \mathbb{R}^n$ and $b, \lambda^* \in R^m$ as follows:

$$\begin{aligned}
x_i^* &= \begin{cases} \text{random positive number} & i = 1, 2, \ldots, m, \\ 0 & i = m+1, m+2, \ldots, n, \end{cases} \\
\mu_i^* &= \begin{cases} \text{random positive number} & i = m+1, m+2, \ldots, n, \\ 0 & i = 1, 2, \ldots, m, \end{cases} \\
\lambda^* &= \text{random vector}, \\
c &= A^\top \lambda^* + \mu^*, \\
b &= Ax^*.
\end{aligned}$$

In doing so, the exact solution is (x^*, λ^*, μ^*). Choose the starting point (x^0, λ^0, μ^0) with the components of x^0 and μ^0 set to large positive values.

Compare the computed numerical solution obtained by the algorithm with the exact solution to verify the correctness of your code. Try different values for m, n and the tolerance ε (`tol`), for example $n = 30, m = 20$ and `tol`$= 10^{-12}$.

Use the provided template `mylinprog.m` as a guideline for the implementation of your algorithm. Print out the value of η in each iteration. Check dimensions of input arguments and give corresponding error messages. Stop the algorithm if `maxiter` is reached and provide the user with a corresponding note. Additionally submit a script file called `mymain.m` to run the above generated example. For comparison, perform several iterations with the function `oneStep` from Exercise sheet 3 with sigma $=$ `1/(k+1)^2`. What do you observe? Comment on your results in the written report.

Listing 1: Template for the MATLAB routine mylinprog.m

```
function [x,lambda,mu] = mylinprog(c,A,b,tol,maxiter, ...
                                                    x0,lambda0,mu0)
%MYLINPROG Linear programming
%  [x,lambda,mu] = MYLINPROG(c,A,b,tol,maxiter,x0,lambda0,mu0)
%  attempts to solve the linear programming problem utilizing
%  the predictor corrector algorithm (Mehrotra):
%
%          min c'*x    subject to    A*x = b, x >= 0
%
%  Input : c ......... vector of dimension nx1
%          A ......... matrix of dimension mxn
%          b ......... vector of dimension mx1
%          tol ....... tolerance for the stopping criteria
%          maxiter ... maximum number of allowed iterations
%          x0 ........ initial guess for the solution x
%                      (dimension nx1)
%          lambda0 ... initial guess for the Lagrange
%                      multiplier lambda (dimension mx1)
%          mu0 ....... initial guess for the Lagrange
%                      multiplier mu (dimension nx1)
%
%  Output: x ......... numerical solution for x
%          lambda .... numerical solution for Lagrange
%                      multiplier lambda
%          mu ........ numerical solution for Lagrange
%                      multiplier mu
%
```