Universität Konstanz
Fachbereich Mathematik und Statistik
Prof. Dr. Stefan Volkwein
Jianjie Lu, Sabrina Rogg

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Numerische Verfahren der restringierten Optimierung

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Program 3 (6 Points)

Submission by E-Mail to jianjie.lu@uni-konstanz.de: 07.02.2017, 10:00 h

Given the nonlinear optimization problem

$$\min \frac{1}{2} x^\top Q x + x^\top d \quad \text{subject to} \quad Ax + F(x) = b,$$

where $Q \in \mathbb{R}^{n\times n}$, $d \in \mathbb{R}^n$, $A = [B|N] \in \mathbb{R}^{m\times n}$ with $B \in \mathbb{R}^{m\times m}$ invertible, $F : \mathbb{R}^n \to \mathbb{R}^m$ a nonlinear function and $b \in \mathbb{R}^m$. Implement the SQP method to solve this problem by following Algorithm 4.3. To solve subproblem (4.5) use the routine `myquadprog` from the first programming exercise. Name your function

```
[x,lambda] = mysqp(Q,d,A,b,Nonlin,x0,lambda0,tol,maxiter).
```

The input parameters `x0, lambda0, tol` and `maxiter` are the same as for the function `mylinprog` from Program 2. The parameter `Nonlin` is a structure containing function handles `F`, `Fp` and `Fpp` in the form:

$$\texttt{Nonlin.F} = F(x), \quad \texttt{Nonlin.Fp} = \nabla F(x) \quad \text{and} \quad \texttt{Nonlin.Fpp} = \lambda^\top \nabla^2 F(x).$$

In case that `Fpp` is not given your program should implement a damped BFGS updating of the form (4.16) to approximate $\nabla_{xx} L(x, \lambda)$, where $B_0 = Q$ is used (*Hint*: Use the commands `fieldnames` and `ismember`).

Test your codes with the following settings:

$$Q = \begin{pmatrix} I & 0 \\ 0 & \nu I \end{pmatrix}, \quad d = \begin{pmatrix} -z \\ 0 \end{pmatrix}, \quad A = \begin{pmatrix} L & -I \end{pmatrix}, \quad b = 0, \quad F(x) = (x_1^3, \ldots, x_m^3)^\top$$

with

$$L = \frac{1}{h^2} \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix} \in \mathbb{R}^{m\times m}, \quad I = \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix} \in \mathbb{R}^{m\times m}$$

and $z_i = \frac{1}{8} \sin(x_i)(17\nu - 60\nu \cos(2x_i) + 3\nu \cos(4x_i) + 8)$ for $x_i = ih$, $i = 1, \ldots, m$ and $h = 2\pi/(m+1)$. Further set $\nu = 10^{-4}$. As a stopping criteria for the SQP method choose $\|\Delta x\|_2 < \texttt{tol}$ with $\texttt{tol} = 10^{-6}$ and set $\texttt{maxiter} = 20$. Try different values for m (i.e. 50, 100, 500, 1000, 1500). Compare the performance of the SQP and the SQP-BFGS implementation. Don't forget to check the dimensions of the input arguments and inform the user if `maxiter` is reached. In the written report give some details on the derivatives.