Universität Konstanz *SS 10*
Fachbereich Mathematik und Statistik
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ÜBUNGEN ZU **Numerik gewöhnlicher Differentialgleichungen**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 1 **Submission: 22.04.2010**

Exercise 1 (Homework) (4 Points)

The method

$$y_{k+1} = y_k + hf\left(x_k + \frac{h}{2}, y_k + \frac{h}{2}f(x_k, y_k)\right), \quad k \geq 0$$

is called *midpoint rule*. Show that this method is of second order.

Exercise 2

Apply the *Euler method*

$$y_{k+1} = y_k + hf(x_k, y_k), \quad k \geq 0$$

to the initial value problem

$$\begin{cases} y'(x) = -y(x) \quad \text{for } 0 \leq x \leq 1, \\ y(0) = 1 \end{cases} \tag{1}$$

with $h = 0.25$. Compare your numerical result with the exact solution. Repeat the computation using the step sizes $h/2$ and $h/4$.

Exercise 3

Apply the *Heun method*

$$y_{k+1} = y_k + \frac{h}{2}\Big(f\big(x_k, y_k\big) + f(x_{k+1}, y_k + hf(x_k, y_k))\Big)$$

to the initial value problem (1) using the step sizes $h = 0.25$, $h/2$ and $h/4$. What kind of results do you expect compared to the results of the Euler method?