*Universität Konstanz* *SS 10*
*Fachbereich Mathematik und Statistik*
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# ÜBUNGEN ZU **Numerik gewöhnlicher Differentialgleichungen**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

**Sheet 2** **Submission: 06.05.2010, 12:00 o'clock, Box 13**

## Exercise 4 (Homework) (4 Points)

Compute the order of consistency for the *modified Euler method (Heun method)*.

## Exercise 5

Consider the scalar initial value problem

$$\begin{aligned} y'(t) &= y(t) - 2\sin t, \quad t \in [0,4] \\ y(0) &= 1. \end{aligned} \tag{1}$$

Show that $y(t) = \sin t + \cos t$ is a solution to (1). Solve the initial value problem (1) by using the *Euler method* and the *modified Euler method*.

| $h$ | $\lvert y_h(1) - y(1)\rvert$ | $\lvert y_h(2) - y(2)\rvert$ | $\lvert y_h(4) - y(4)\rvert$ |
|---|---|---|---|
| $2^{-4}$ | | | |
| $2^{-5}$ | | | |
| $2^{-6}$ | | | |
| $2^{-7}$ | | | |

Tabelle 1: Numerical solution ($y_h$) compared with exact solution ($y$).

Write down the iteration update for both methods and fill out Table 1. Document your observations when comparing these two methods.

## Exercise 6

Apply the *trapezoidal method* to the initial value problem (1). Write down the iteration update and fill out Table 1. What do you observe compared to the results of the methods used in Exercise 5?

**Program 1** (10 Points)

Consider the following nonlinear problem

$$\begin{aligned} \dot{x}(t) &= \alpha x(t) + \beta x(t) y(t) \\ \dot{y}(t) &= \gamma y(t) + \delta x(t) y(t), \quad t > 0 \end{aligned} \tag{2}$$

together with the initial conditions

$$x(0) = x_0 \qquad \text{and} \qquad y(0) = y_0,$$

where $\alpha > 0$, $\beta < 0$, $\gamma < 0$ and $\delta > 0$. A stationary point for (2) is given by

$$x_s = -\frac{\gamma}{\delta} \qquad \text{and} \qquad y_s = -\frac{\alpha}{\beta}.$$

In a neighborhood of $(x_s, y_s)$ the form of the solution curve $\{(x(t), y(t))\}_{t>0}$ is close to an ellipse.
We choose

$$\alpha = \frac{1}{4}, \quad \beta = -\frac{1}{100}, \quad \gamma = -1, \quad \delta = \frac{1}{100},$$

and $x_0 = 80$, $y_0 = 30$.

1. Compute a numerical solution to (2) using the Euler and the Heun methods with the step sizes $h = 1$, $h = 0.5$ and $h = 0.25$ and visualize the results.

2. How small a step size do you have to use for the graph of the solution to close back on itself to visual accuracy?

3. Repeat part 1 by using the *fourth-order Runge-Kutta method*

$$z_{k+1} = z_k + \frac{h}{6}(F_1 + 2(F_2 + F_3) + F_4), \quad k \geq 0$$

   with $z_k = (x_k, y_k)$ and

$$\begin{aligned} F_1 &= f(t_k, z_k), \\ F_2 &= f\left(t_k + \frac{h}{2}, z_k + \frac{h}{2} F_1\right), \\ F_3 &= f\left(t_k + \frac{h}{2}, z_k + \frac{h}{2} F_2\right), \\ F_4 &= f(t_k + h, z_k + h F_3). \end{aligned}$$

4. Test the stability of the solution to (2) with respect to changes in the initial conditions by changing $x_0 = 80$, $y_0 = 30$ by a unit amount in each direction (four different cases) and repeat the calculations from part 3.