Universität Konstanz *SS 10*
Fachbereich Mathematik und Statistik
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Übungen zu **Numerik gewöhnlicher Differentialgleichungen**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 5 **Submission: 17.06.2010, 10:00 o'clock, IN CLASS**

Exercise 13 (Homework)

(4 Points)

Consider the method

$$y_{n+1} = y_{n-1} + 2hf(t_n, y_n) \tag{1}$$

and apply it on the initial value problem

$$y' = -2y + 1, \quad y(0) = 1.$$

Using the theory of the difference equations, analyze the behavior of the sequence $\{y_n\}$ as $n \to \infty$.

Exercise 14

Solve the difference equation

$$y_{n+1} = \frac{5}{2}y_n + y_{n-1}, \quad y_0 = y_1 = 1,$$

in terms of the roots of its characteristic equation. Discuss the behavior of the sequence $\{y_n\}$ as $n \to \infty$.

Exercise 15

Verify that method (1) of Exercise 13 is second-order accurate.

Program 4 (10 Points)

Implement the two functions

`[y,t] = odesolveABM3(fun,y0,t,h)` and `[y,t] = odesolveAB4(fun,y0,t,h)`

for solving ordinary differential equations and systems of ordinary differential equations, where `fun` is a function handle to a function of the form `my_fun(t,y)`, `y0` an initial condition, `t = [t0,T]` with `t0` the initial time and `T` the end time and `h` the step size. The return values are the solution `y` and the time grid `t`.

- `odesolveABM3` is a predictor-corrector method consisting of a *third-order Adams Bashforth method* (predictor) and a *third-order Adams-Moulton method* (corrector) given as

$$y^{j+1,0} = y^j + \frac{h}{12}\big(23f(t_j,y^j) - 16f(t_{j-1},y^{j-1}) + 5f(t_{j-2},y^{j-2})\big)$$
$$y^{j+1,1} = y^j + \frac{h}{24}\big(9f(t_{j+1},y^{j+1,0}) + 19f(t_j,y^j) - 5f(t_{j-1},y^{j-1}) + f(t_{j-2},y^{j-2})\big)$$
$$y^{j+1} := y^{j+1,1}.$$

- `odesolveAB4` is a *fourth-order Adams Bashforth method* given as

$$y^{j+1} = y^j + \frac{h}{24}\big(55f(t_j,y^j) - 59f(t_{j-1},y^{j-1}) + 37f(t_{j-2},y^{j-2}) - 9f(t_{j-3},y^{j-3})\big).$$

The inital values y^1, y^2 and y^3 should be computed with a fourth-order Runge-Kutta method. Apply the two solvers to the initial value problem

$$y'(t) = \lambda y(t) - (\lambda+1)e^{-t}\,, \quad t \in [0,2]\,, \quad y(0) = 1,$$

with a constant $\lambda < 0$. The solution to this problem is given by $y(t) = e^{-t}$ (independent of λ). Fill out Table 1 with $|y_h(2) - e^{-2}| = |y^{2/h} - e^{-2}|$, generate some plots and interpret the obtained results. What can you say about the order of consistency when looking at the numerical results.

	$\lambda = -2, \lvert y^{2/h} - e^{-2}\rvert$		$\lambda = -20, \lvert y^{2/h} - e^{-2}\rvert$	
h	`odesolveAB4`	`odesolveABM3`	`odesolveAB4`	`odesolveABM3`
2^{-3}				
2^{-4}				
2^{-5}				
2^{-6}				
2^{-7}				

Table 1: Numerical solution $y^{2/h}$ compared with exact solution e^{-2}.

Note that the functions `odesolveABM3` and `odesolveAB4` should not open any figure. The code should be well documented and a function description/usage should be accessible by the `help` command.