Universität Konstanz *SS 10*
Fachbereich Mathematik und Statistik
S. Volkwein, O. Lass, R. Mancini

Übungen zu **Numerik gewöhnlicher Differentialgleichungen**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 6 **Submission: 01.07.2010, 12:00 o'clock, Box 13**

Exercise 16 (Homework) (4 Points)

Consider the nonlinear boundary value problem

$$v''(x) = 3v(x) + x^2 + 10v^3(x)\,, \quad v(0) = v(1) = 0\,, \quad 0 < x < 1. \tag{1}$$

a) Discretize (1) using central differences and write down the system for the n approximations v_i of the solution v to (1) at the inner grid points $x_i = i/(n+1)$ for $i = 1, \dots, n$.

b) To solve the nonlinear system we want to apply the *Newton method*. Compute the Jacobian matrix for the discretized system obtained in part a).

c) Write down the *Newton iteration* as a pseudocode.

Exercise 17

Write down the *implicit Euler* and *trapezoidal* method for

$$y'(t) = \lambda y(t) \quad \text{with} \quad \lambda < 0.$$

Further, represent the update using the stability function. What can you say about the damping when comparing the two methods.

Exercise 18

Given the initial value problem

$$y'''(t) + y'(t) = ty(t) \text{ for } t > 2\,, \quad y(2) = 0\,, \quad y'(2) = 2\,, \quad y''(2) = 2. \tag{2}$$

a) Transform (2) into a system of ordinary differential equations.

b) Compute the numerical solution of the transformed system at $t = 2.5$ with one step of the *implicit Euler method* .

Program 5 (10 Points)

In this programming homework we will focus on the use of the *Newton method* for solving nonlinear initial value and boundary value problems.

a) Implement a program to solve the nonlinear ordinary differential equation

$$y'(t) = t\,\sin(y(t))\,, \quad y(0) = y_0\,, \quad t \in [0,5]$$

using the trapezoidal method. Further, use the *Newton method* to solve the nonlinear problem arising from the trapezoidal rule. When using the Newton method solve to an accuracy of 10^{-6}, i.e. $|\delta| < 10^{-6}$, where δ is the Newton update.

As initial value y_0 choose the values $0.5, 1, \ldots, 4.5$ and for the step size choose $h = 0.1$. Plot all results into one figure and describe what you observe. Don't forget to label your axis, add a title and provide a legend to the plot for better understanding.

To verify your obtained results solve the equation with the MATLAB inbuilt solver `ode45`. Generate the same plot as before. Again add labels to the axis, add a title and provide a legend to the plot for better understanding.

In your written report provide a description on how you get the trapezoidal update and how you apply the Newton method together with the interpretation of your obtained results.

b) Implement a program to solve the nonlinear boundary value problem (1). Apply the *Newton method* to solve the problem and use the findings of Exercise 16. When using the Newton method solve to an accuracy of 10^{-6}, i.e. $|\delta| < 10^{-6}$, where δ is the Newton update.

Run your code for $n = 10^i - 1$, $i = 1, 2, 3$ and compare the results. Generate a plot including the boundaries. Don't forget to label your axis, add a title and provide a legend to the plot for better understanding.