Universität Konstanz *WS 12/13*
Fachbereich Mathematik und Statistik
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ÜBUNGEN ZU **Numerische Verfahren der restringierten Optimierung**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 1 **Submission: 12.11.2012, 9:45 o'clock**

Exercise 1 (Homework) (2 Points)

Consider the problem of finding the point on the parabola $y = \frac{1}{5}(x-1)^2$ that is close to $(x, y) = (1, 2)$, in the Euclidean norm sense. We can formulate this as

$$\min f(x, y) = (x-1)^2 + (y-2)^2 \quad \text{u.d.N. } (x-1)^2 = 5y.$$

a) Find all the KKT points for this problem. Are all points regular points?

b) Which of these points are solutions?

c) By directly substituting the constraint into the objective function and eliminating the variable x, we obtain an unconstrained optimization problem. Show that the solutions of this problem cannot be solutions of the original problem.

Exercise 2

Solve the problem

$$\min_x x_1 + x_2 \quad \text{s.t. } x_1^2 + x_2^2 = 1$$

by eliminating the variable x_2. Show that the choice of sign for the square root opertaion during the elimination process is critical; the "wrong" choice leads to an incorrect answer.

Exercise 3

Consider the problem

$$\min_x \left(x_1 - \frac{3}{2}\right)^2 + (x_2 - t)^4 \quad \text{s.t.} \begin{bmatrix} 1 - x_1 - x_2 \\ 1 - x_1 + x_2 \\ 1 + x_1 - x_2 \\ 1 + x_1 + x_2 \end{bmatrix} \geq 0,$$

where t is a parameter to be fixed prior to solving the problem.

a) For what values of t does the point $x^* = (1, 0)^\top$ satisfy the KKT conditions?

b) Show that when $t = 1$, only the first constraint is active at the solution, and find the solution.