Universität Konstanz *WS 12/13*
Fachbereich Mathematik und Statistik
S. Volkwein, O. Lass, S. Trenz, R. Mancini

Übungen zu **Numerische Verfahren der restringierten Optimierung**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 4 **Submission: 07.01.2013, 9:45 o'clock**

Exercise 10 (Homework) (2 Punkte)

Let A be the matrix in the equality constraint given by $[B|N]$, with B invertible. Show that the basis matrices

$$Y = \begin{bmatrix} B^{-1} \\ 0 \end{bmatrix} \quad \text{and} \quad Z = \begin{bmatrix} -B^{-1}N \\ I \end{bmatrix}$$

are linearly independent and that the assumptions for the *null space method* are satisfied. Furthermore, let $x = [x_B \; x_N]^\top$. Write the optimization problem

$$\begin{aligned} &\min \sin(x_3 + x_4) + x_1^2 + \tfrac{1}{3}(x_5 + x_6^4 + x_2/2) \\ &\text{subject to} \quad x_1 + 8x_3 - 6x_4 + 9x_5 + 4x_6 = 6 \\ &\qquad\qquad\quad 4x_2 + 3x_3 + 2x_4 - x_5 + 6x_6 = -4 \end{aligned}$$

in reduced form by using the matrices Y and Z. Show first that $x_B = B^{-1}b - B^{-1}Nx_n$ is satisfied.

Exercise 11

Compute the inverse of the KKT-Matrix (3.1).

Exercise 12

The problem of finding the shortest euclidean distance from a point x_0 to the hyperplane $\{x \mid Ax = b\}$, where A has full row rank, can be formulated as a quadratic program. Write the problem in the form ($\mathbf{QP}_{Gl}$), derive the KKT-system (3.2) and determine the solutions x^* and λ^* explicitly. Further, show that in the special case in which A is a row vector, the shortest distance from x_0 to the solution set of $Ax = b$ is $|b - Ax_0|/\|A\|_2$.

Merry Christmas and a happy new year!

Buon Natale e felice anno nuovo!