Universität Konstanz *WS 12/13*
Fachbereich Mathematik und Statistik
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ÜBUNGEN ZU **Numerische Verfahren der restringierten Optimierung**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 5 **Submission: 21.01.2013, 9:45 o'clock**

Note: The 3. Program is already available online with submission on Monday, January 28, 9:45 o'clock!

Exercise 13 (Homework) (2 Points)

Let $\bar{x} \in \mathbb{R}^n$ be given, and let x^* be the solution of the projection problem

$$\min \|x - \bar{x}\|^2 \quad \text{subject to} \quad l \leq x \leq u.$$

For simplicity, assume that $-\infty < l_i < u_i < \infty$ for all $i = 1, 2, \ldots, n$. Show that the solution of this problem coincides with the projection formula given by

$$P(x, l, u)_i = \begin{cases} l_i & \text{if} \quad x_i < l_i, \\ x_i & \text{if} \quad x_i \in \ [l_i, u_i], \\ u_i & \text{if} \quad x_i > u_i, \end{cases}$$

that is, show that $x^* = P(\bar{x}, l, u)$.

Exercise 14

Write down KKT conditions for the following convex quadratic program with mixed equality and inequality constraints:

$$\min q(x) = \frac{1}{2} x^\top Q x + x^\top c \quad \text{subject to} \quad Ax \geq b, \quad \bar{A}x = \bar{b},$$

where Q is symmetric and positive semidefinite. Further, modify the interior point method from the lecture to solve this problem.

Exercise 15

Consider the quadratic optimization problem given by

$$\min \ f(x) := \frac{1}{2} x^\top Q x + x^\top d + c,$$

where $Q \in \mathbb{R}^{n \times n}$ is symmetric and positive definite. Let x^* be the minimizer of f and define the energy norm as $\|x\|_Q := (x^\top Q x)^{1/2}$. Show that the following condition holds:

$$f(x) = \frac{1}{2} \|x - x^*\|_Q^2 + f(x^*).$$