Universität Konstanz *WS 12/13*
Fachbereich Mathematik und Statistik
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ÜBUNGEN ZU **Numerische Verfahren der restringierten Optimierung**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 6 **Submission: 04.02.2013, 9:45 o'clock**

Exercise 16 (Homework) (2 Points)

Given the problem

$$\min_{x\in\mathbb{R}^2} -x_1 - x_2 \quad \text{s.t.} \quad g(x) := -x \leq 0,\ e(x) := x_1^2 + x_2^2 - 1 = 0. \tag{1}$$

a) Sketch the admissible set and the cost function $f(x)$. (Use contour lines for the cost function.)

b) Determine the solution of (1) and the corresponding Lagrange multipiers.

c) Let $x^k = (-1/2, -1/2)^\top$ be given. Sketch the constraints of the SQP subproblem and show that the corresponding admissible set is empty.

Exercise 17

Prove the following statement:

Let f and e be twice continuously differentiable and let (x^*, λ^*) be such that

a) $\nabla e_1(x^*), \dots, \nabla e_m(x^*)$ are linearly independent

b) $s^\top \nabla_{xx} L(x^*, \lambda^*) s > 0$ holds for all $s \in \mathbb{R}^n \setminus \{0\}$ with $A(x^*)s = 0$.

Then $F'(x^*, \lambda^*)$ is regular, where

$$F'(x, \lambda) = \begin{pmatrix} \nabla^2_{xx} L(x, \lambda) & A^\top(x) \\ A(x) & 0 \end{pmatrix}.$$

Exercise 18

Given the problem

$$\min f(x) \quad \text{s.t.} \quad e(x) = 0, \tag{2}$$

where f and e are C^2 functions. The *augmented Lagrange function* for (2) is defined as

$$L_\alpha(x, \lambda) := f(x) + \lambda^\top e(x) + \frac{\alpha}{2}\|e(x)\|^2$$

with $\alpha \geq 0$. Consider the example $f(x) = -x_1 x_2^2$ and $e(x) = x_1^2 + x_2^2 - 1$. Determine the KKT pairs (x^*, λ^*) that satisfy the second order sufficient optimality condition. Further show that for sufficiently large α,

$$\nabla L_\alpha(x, \lambda) = 0 \quad \text{and} \quad \nabla_{xx} L_\alpha(x, \lambda) \quad \text{positive definite}$$

holds.