Universität Konstanz *WS 10/11*
Fachbereich Mathematik und Statistik
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ÜBUNGEN ZU **Theorie und Numerik partieller Differentialgleichungen**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 3 **Submission: 10.01.2011, 10:30 o'clock, Box 18**

Exercise 7 (4 Points)

Let $A \in \mathbb{R}^{(M-1)^2 \times (M-1)^2}$ be the matrix obtained by the classical finite difference method for solving the boundary value problem $-\Delta u = g$ in $\Omega = (0,1) \times (0,1)$ and $u = \gamma$ on $\partial\Omega$. Show that the vectors $u^{kl} \in \mathbb{R}^{(M-1)^2}$ with stepsize $h = 1/M$,

$$(u^{kl})_{ij} = \sin\left(\frac{ik\pi}{M}\right) \sin\left(\frac{jl\pi}{M}\right), \quad 1 \le i,j \le M-1$$

are the eigenvectors of A. What are the corresponding eigenvalues λ_{kl}?

Exercise 8 (4 Points)

Given the problem

$$\begin{aligned} -\Delta v &= \lambda v \text{ in } \Omega, \\ v|_{\partial\Omega} &= 0, \end{aligned} \tag{1}$$

$\Omega \subset \mathbb{R}^2$ a bounded domain with piecewise smooth boundary $\partial\Omega$. A solution $v \in C^2(\Omega) \cap C(\bar{\Omega}), v \neq 0$ is called an eigenfunction to the eigenvalue λ.

a) Show that all eigenvalues λ of (1) are positive.

b) Let v_1, v_2 be eigenfunctions to the corresponding eigenvalues λ_1 and λ_2 with $\lambda_1 \neq \lambda_2$. Show that v_1, v_2 are orthogonal in the associated inner product

$$\langle u, w \rangle = \int_\Omega u(x)w(x)dx.$$

c) Compute the eigenvalues and eigenfunctions of (1) in the case $\Omega = (0,1) \times (0,1)$ and compare them with the results of Exercise 7.

Exercise 9 (4 Points)

Given the problem

$$-\Delta u = 1 \text{ in } \Omega = (0,1) \times (0,1), \quad u = 0 \text{ on } \partial\Omega. \tag{2}$$

Make a Ritz-Ansatz with the Ansatzfunctions

$$u^{kl}(x,y) = \sin(k\pi x)\sin(l\pi y), \; (x,y) \in \Omega, \; l = 1, \dots, \hat{l}, \; k = 1, \dots, \hat{k}.$$

What solution do you obtain for u?