Universität Konstanz *WS 10/11*
Fachbereich Mathematik und Statistik
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Übungen zu **Theorie und Numerik partieller Differentialgleichungen**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 5 **Submission: 24.01.2011, 11:00 o'clock, Box 18**

Exercise 13 (4 Points)

Let $\Omega = (-1,1)$ and let $u(x) = |x|$, $x \in \Omega$. Show that the weak derivative of u is given by

$$u'(x) = \begin{cases} -1, & \text{if} \quad -1 < x < 0, \\ 1, & \text{if} \quad 0 < x < 1, \end{cases}$$

and $u' \in L^2(\Omega)$. Further show that u' is not weakly differentiable.

Exercise 14 (4 Points)

Compute the piecewise quadratic finite element basis functions on the reference triangle $(0,0)$, $(1,0)$, $(0,1)$ corresponding to the corner point $(0,1)$ and the edge point $(0,0.5)$. Visualize these two quadratic basis functions using Matlab.

Hint: For the visualization one can use the commands `initmesh, refinemesh` and `pdesurf` together with the provided geometry file `triangle.m`.

Exercise 15 (4 Points)

We consider the following system of ordinary differential equations

$$\begin{aligned} M\dot{x}(t) &= Ax(t) + b(t), \quad t \in (0,T], \\ x(0) &= x_0, \end{aligned} \tag{1}$$

where M, A are real $n \times n$ matrices satisfying $\det M \neq 0$, $A = A^\top$, $M = M^\top$, M positive definite, A negative semidefinite, $T > 0$ is the terminal time, $b : [0,T] \to \mathbb{R}^n$ is a continuous inhomogeneity and $x_0 \in \mathbb{R}^n$ is a given initial condition.

a) Formulate the explicit and implicit Euler methods for (1).

b) Give the trapezoidal method for (1).

c) Discuss if all three methods are well-defined.

Hint: The explicit and implicit Euler as well as the trapezoidal method are explained in the lecture notes *Numerik gewöhnlicher Differentialgleichungen* by Prof. S. Volkwein.