Universität Konstanz *WS 10/11*
Fachbereich Mathematik und Statistik
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Übungen zu **Theorie und Numerik partieller Differentialgleichungen**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Sheet 6 **Submission: 02.02.2011, 11:00 o'clock, Box 18**

Exercise 16 (4 Points)

Compute the stiffness matrix for a one dimensional problem with the following basis functions:

1. Piecewise linear and globally continuous nodal basis functions
2. Polynomial basis functions (i.e. $X_h = span\{x, x^2, ..., x^N\}$)

Further compute the condition number of each stiffness matrix. What do you observe when different values for N are used (for example $N = 5, 10, 15$)?

Hint: The condition number of a symmetric positive definite matrix S is given by

$$\kappa(S) = \frac{\lambda_{\max}}{\lambda_{\min}}.$$

One can compute the eigenvalues by using the Matlab command `eig`.

Exercise 17 (4 Points)

Given the Poisson problem

$$\begin{aligned} -\Delta u(x,y) &= f(x,y) \qquad && (x,y) \in \Omega, \\ u(x,y) &= g(x,y) \qquad && (x,y) \in \partial\Omega \end{aligned} \tag{1}$$

with

$$\begin{aligned} f(x,y) &= 4\pi \sin(2\pi x)(\pi \cos(2\pi y^2)(1+4y^2) + \sin(2\pi y^2)), \\ g(x,y) &= \sin(2\pi x)\cos(2\pi y^2) \end{aligned}$$

and $\Omega = (0,1) \times (0,1)$. Solve (1) by using the Partial Differential Equation Toolbox in Matlab using the graphical user interface `pdetool`. Follow the steps **Draw**, **Boundary**, **PDE**, **Mesh** and **Solve**. Finally draw the solution.

Hint: A short and good summary on the use of the Partial Differential Equation Toolbox can be found in the web by Prof. Heinrich Voss with the name *Eine sehr kurze Einführung in die Partial Differential Equation Toolbox von MATLAB.*

Exercise 18 (4 Points)

Let $\Omega = (0,1) \subset \mathbb{R}$. Given the heat equation

$$\begin{aligned} \frac{\partial u(x,t)}{\partial t} - \Delta u(x,t) + b\nabla u(x,t) + cu(x,t) &= f(x) \qquad && (x,t) \in (0,1) \times (0,T), \\ u(x,0) &= u_0 && x \in (0,1), \\ u(0,t) = u(1,t) &= 0 && t \in (0,T). \end{aligned} \tag{2}$$

Discretize the problem using finite differences and implicit time steps (compare to Exercise 15). For the term ∇u use the upwind method as introduced in Exercise 2. Write down (2) in the same form as in Exercise 15 and give matrices M and A explicitly.