Universität Konstanz *WS 10/11*
Fachbereich Mathematik und Statistik
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ÜBUNGEN ZU Theorie und Numerik partieller Differentialgleichungen

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Submission: 20.12.2011, 11:00 o'clock
Codes by E-Mail and Reports in Box 18

Program 1 (8 Points)

Let $\Omega =]0,1[^2$ and $h = \frac{1}{M}$ with $M \in \mathbb{N}$. Solve numerically the Poisson problem

$$\begin{aligned} -\Delta u(x,y) &= f(x,y), \qquad (x,y) \in \Omega \\ u(x,y) &= g(x,y), \qquad (x,y) \in \partial\Omega \end{aligned}$$

with the classical finite difference method (i.e. five-point-stencil). Use the lexicographical ordering of the grid points in Ω. Use the following functions f and g:

a) $f(x,y) = 4\pi \sin(2\pi x)(\pi \cos(2\pi y^2)(1+4y^2) + \sin(2\pi y^2))$,
$g(x,y) = \sin(2\pi x)\cos(2\pi y^2)$.

b) $f(x,y) = \begin{cases} 1, \text{ if } |x-0.5| + |y-0.5| \le 0.2, \\ 0, \text{ otherwise.} \end{cases}$
$g(x,y) = 0$.

c) $f(x,y) = 0$,
$g(x,y) = \begin{cases} 1, \text{ if } |x| \le 0.5, \\ 0, \text{ otherwise.} \end{cases}$

Show that in case a) $u(x,y) = g(x,y)$, $(x,y) \in \Omega$ holds, and use this property to check your code for correctness. Visualize the right hand side $f(x,y)$ and the numerical solution $u(x,y)$. Don't forget to lable the plots (`title, xlabel, ylabel, zlabel, ...`). Try different values for M. Document your code well and write a report including your observations.

Remark: For the MATLAB[1] implementation the following commands can be useful: `ndgrid, mesh, spdiags, sparse`.

[1] Alternatively OCTAVE can be used for the implementation.