Universität Konstanz *WS 10/11*
Fachbereich Mathematik und Statistik
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ÜBUNGEN ZU **Theorie und Numerik partieller Differentialgleichungen**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Submission: 02.02.2011, 11:00 o'clock
Codes by E-Mail and Reports in Box 18

Program 3

(8 Points)

Let $\Omega \subset \mathbb{R}^2$. Solve numerically the parabolic partial differential equation

$$\begin{aligned} \frac{\partial u(\mathbf{x},t)}{\partial t} - \operatorname{div}(c(\mathbf{x})\nabla u(\mathbf{x},t)) + a(\mathbf{x})u(\mathbf{x},t) &= 0 \qquad \text{in} \quad \Omega \times (0,T), \\ u(\mathbf{x},0) &= u_0(\mathbf{x}), \\ \frac{\partial u(\mathbf{x},t)}{\partial n} &= 0, \qquad \text{on} \quad \partial\Omega \times (0,T) \end{aligned} \tag{1}$$

applying the implicit Euler method introduced in Exercise 15, Sheet 5. For the domain Ω we choose $B_1(0,0)$ (i.e. disc with radius 1 and center $(0,0)$). Further we set $a(\mathbf{x}) = 3$ and $c(\mathbf{x}) = 1/8$. As an initial value we choose

$$u_0(\mathbf{x}) = \begin{cases} 1, & \text{if } |x^2 + y^2| \leq 0.1, \\ 0, & \text{otherwise.} \end{cases}$$

Compute the numerical solution to (1) in the time interval $(0,1)$ using 20 timesteps. For the spacial discretization we use the finite element discretization provided by the PARTIAL DIFFERENTIAL EQUATION TOOLBOX in MATLAB. Use about 2000 finite elements. Visualize the numerical solution $u(\mathbf{x},t)$ in a good way. Do not forget to label the plots.

Hint: For the implementation the commands `initmesh`, `refinemesh`, `assempde`, `assema`, `pdesurf` together with the provided geometry and boundary files `circle11_geom.m` and `circle11_bdry.m` can be useful.