

ÜBUNGEN ZU Theorie und Numerik partieller Differentialgleichungen

<http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/>

Deadline: 2012/01/09, 9:00 a.m.
Codes and Reports by E-Mail

Program 1

(8 Points)

Let $\Omega =]0, 1[^2$. For each direction x and y consider $n \in \mathbb{N}$ the number of inner discretization points with corresponding step size $h = \frac{1}{n+1}$, i.e., there is a total number of $n + 2$ discretization points in each direction. Solve numerically the Poisson problem

$$\begin{aligned} -\Delta u(x, y) &= f(x, y), & (x, y) \in \Omega \\ u(x, y) &= g(x, y), & (x, y) \in \partial\Omega \end{aligned}$$

with the classical finite difference method (i.e. five-point-stencil). Use the lexicographical ordering of the grid points in Ω . Use the following functions f and g :

a) $f(x, y) = 4\pi \sin(2\pi x)(\pi \cos(2\pi y^2)(1 + 4y^2) + \sin(2\pi y^2)),$

$$g(x, y) = \sin(2\pi x) \cos(2\pi y^2).$$

b) $f(x, y) = \begin{cases} 1, & \text{if } |x - 0.5| + |y - 0.5| \leq 0.2, \\ 0, & \text{otherwise.} \end{cases}$

$$g(x, y) = 0.$$

c) $f(x, y) = 0,$

$$g(x, y) = \begin{cases} 1, & \text{if } |x| \leq 0.5, \\ 0, & \text{otherwise.} \end{cases}$$

Show that in case a) $u(x, y) = g(x, y)$, $(x, y) \in \Omega$ holds, and use this property to check your code for correctness. Visualize the right hand side $f(x, y)$ and the numerical solution $u(x, y)$. Don't forget to lable the plots (`title`, `xlabel`, `ylabel`, `zlabel`, ...). Try different values for n . Document your code well and write a report including your observations.

Remark: For the MATLAB implementation the following commands can be useful: `ndgrid`, `mesh`, `spdiags`, `sparse`.