Universität Konstanz *WS 11/12*
Fachbereich Mathematik und Statistik
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Übungen zu **Theorie und Numerik partieller Differentialgleichungen**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Deadline: 03.02.2012, 18:00 o'clock
Codes and Reports by E-Mail to the tutors

Program 3 (8 Points)

Let $\Omega \subset \mathbb{R}^2$. Solve numerically the parabolic partial differential equation

$$\begin{aligned} \frac{\partial y(\mathbf{x},t)}{\partial t} - \operatorname{div}(c(\mathbf{x})\nabla y(\mathbf{x},t)) + a(\mathbf{x})y(\mathbf{x},t) &= f(\mathbf{x},t) \quad \text{in} \quad \partial\Omega \times (0,T), \\ y(\mathbf{x},0) &= y_0(\mathbf{x}) \quad \text{in} \quad \Omega, \\ y(\mathbf{x},t) &= 0 \quad \text{on} \quad \partial\Omega \times (0,T) \end{aligned} \tag{1}$$

by applying the implicit Euler method introduced in Exercise 12, Sheet 4. For the spatial discretization we use the finite element discretization provided by the Partial Differential Equation Toolbox in Matlab. Write therefore a function

```
[Y,p,e,t] = fem_parabolic_2D(g,b,Nref,T0,Tf,Nt,c,a,f,y_init)
```

with the input parameters

- `g`, `b` the given geometry and boundary files "`program3_geometry.m`" and "`program3_boundary.m`",
- `Nref` the number of mesh refinements,
- `T0`, `Tf` initial and final time,
- `Nt` the number of time steps,
- `c`, `a`, `f` the parameters and the right-hand side of the parabolic equation,
- `y_init` the initial function $y_0(\mathbf{x})$,

and the output parameters

- `Y` the solution matrix
- `p`, `e`, `t` the refined mesh data (see therefore the Matlab documentation for `initmesh` and `refinemesh`).

Call the function from a `main.m`-file.

Visualize the numerical solution $y(\mathbf{x}, t)$ in an appropriate way (e.g. a movie). **Do not create a new figure-object for each time step!** Do not forget to label the plots.

For the coefficient functions in the formulation set $c(\mathbf{x}) = \frac{1}{8}$ and $a(\mathbf{x}) = 0$, and $f(\mathbf{x}, t) = 0$ for the right-hand side. As initial condition choose:

$$y_0(\mathbf{x}) = \begin{cases} 1, & \text{if } |x_1^2 + x_2^2| \leq 0.1, \\ 0, & \text{otherwise.} \end{cases}$$

Compute the numerical solution to (1) on the time interval $(0, 1)$ with `Nt=20` time steps. For the spatial domain Ω we consider $B_1(0, 0)$ (i.e. the unit circle with radius 1 and center $(0, 0)$), which is given by the geometry-file `program3_geometry.m`. Use about 2000 finite elements (i.e. `Nref=2`).

Guideline

Use the MATLAB Partial Differential Equation Toolbox functions `initmesh`, `refinemesh`, `assempde`, `assema`, `pdesurf`. Read therefore the documentation.
In particular we recommend to use the following `assempde`-function

```
[K,F] = assempde(b,p,e,t,c,a,f)
```

where the matrix `K` can be considered as the matrix Ψ from Exercise 12. For assembling the mass matrix for the parabolic part use the `assema`-function with following parameters:

```
[K,M,F] = assema(p,t,0,1,0),
```

where the matrix `M` can be considered as Φ. Make sure that you do not overwrite the `K` and `F` you calculated before.

Use the help function by typing "`help <functionname>`" in the MATLAB console for getting important information about all functions you do not know!