Universität Konstanz
FB Mathematik & Statistik
Dr. Z. Yang

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# Advection-Diffusion-Reaction Problems

Blatt 4

## Patterns of Animal Coats

J. D. Murray suggests that a single mechanism could be responsible for generating all of the common animal patterns observed. This mechanism is based on a reaction-diffusion system of the morphogen prepatterns, and the subsequent differentiation of the cells to produce melanin simply reflects the spatial patterns of morphogen concentration,

$$\begin{aligned} u_t &= u_{xx} + u_{yy} + \lambda f(u, v), \\ v_t &= d(v_{xx} + v_{yy}) + \lambda g(u, v), \quad t > 0, \quad (x, y) \in \Omega \subset \mathbb{R}^2, \end{aligned} \tag{1}$$

where $d$ and $\lambda$ are constant and stand for the diffusion and reaction coefficients. $f(u, v)$ and $g(u, v)$ are given reaction functions.

(i) Please refer to a useful website 'PDEs and Animal Patterns' (lecture 6, Turing Patterns in animal coats) which shows the basic idea of J. D. Murray's theory. The more detailed description is given in his book, chapter 14 and 15. Be ready to give a brief explanation of this theory.

(ii) Take $\Omega = [0, a] \times [0, b]$ and $d = 100$. The reaction functions are given in the form of

$$\begin{aligned} f(u, v) &= 0.5 - u + u^2 v, \\ g(u, v) &= 1 - u^2 v. \end{aligned} \tag{2}$$

Periodic boundary condition is used at $x = 0, a$ and Neumann condition at $y = 0, b$.

Apply first order Euler forward discretization to the time derivative and the central difference to the spacial derivative, treat reaction terms explicitly and diffusion terms implicitly (a useful slide is given online), and make the corresponding Matlab implementation to show the numerical result.