Universität Konstanz
FB Mathematik & Statistik
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Ausgabe: 15. Mai; SS09

Advection-Diffusion-Reaction Problems

Blatt 2

Finite Difference Methods for Typical Advection Equation and Diffusion Equation

Consider a constant coefficient advection equation

$$\begin{aligned} &u_t + au_x = 0, \quad t \geq 0, \quad x \in \mathbb{R}, \\ &u(0,x) = u_0(x), \end{aligned} \tag{1}$$

and a diffusion equation

$$\begin{aligned} &u_t = du_{xx}, \quad t \geq 0, \quad x \in \mathbb{R}, \\ &u(0,x) = u_0(x). \end{aligned} \tag{2}$$

(i) Please give their exact solutions and describe their behavior in $t - x$ space. Fox example, take $u_0(x) = \exp(-x^2)$.

(ii) Consider $x \in [0,1]$, $u_0(x) = (\sin(\pi x))^{100}$. Apply the Euler forward discretization for the temporal derivative, the up-wind discretization for the advection term and the central difference for the diffusion term, build up finite difference algorithms for equations (1) and (2) respectively, implement the correspondent matlab programs, display the numerical solutions at different time levels and compare the results with the exact solution.

(iii) Analyze the consistency, stability and convergence of the above algorithm, use modified equations to explain the artificial diffusion, dispersion and dissipation.