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Advection-Diffusion-Reaction Problems

Blatt 3

Finite Difference Methods for Typical Reaction Problems

Consider the two-way chemical reaction problem (Hundsdorfer & Verwer : Numerical Solution ..., Chap. 1, part1, example 2.3 in page 23),

$$\begin{aligned} w_1'(t) &= -k_1 w_1(t) + k_2 w_2(t), \\ w_2'(t) &= k_1 w_1(t) - k_2 w_2(t), \end{aligned} \tag{1}$$

with reaction constants $k_2 >> k_1$.

(i) Apply the Euler forward and backward discretizitions for the temporal derivative, implement the correspondent matlab programs, display the numerical solutions at different time levels and compare the results with the exact solution.

(ii) Be ready to explain the stiffness of the problem and the stability of the explicit and implicit schemes.

Method of Line for Advection-Diffusion Problems

Consider a simple advection-diffusion equation

$$u_t + a u_x = d u_{xx} \tag{2}$$

with constant coefficients a and d. Please using up-wind advection discritezition and the central difference for the diffusion term to build up a semi-discrete systems of ODEs, then apply IMEX Runge-Kutta method (for example using implicit and explicit trapezoidal rule) and implement the related Matlab programs. Study and explain the numerical behavior and the stability for the advection dominated case $a >> d$ and diffusion dominated case $d >> a$.

Useful reference can be found in the book: Hundsdorfer & Verwer : Numerical Solution ..., Chap. 1: part2 and part 3, Chap 4 : IMEX methods.