



Advection-Diffusion-Reaction Problems

Blatt 1

1. Model derivation

Chemo-taxis problems from mathematical biology are very interesting examples, which take the form of advection diffusion reaction (bio-chemistry occurs) systems,

$$\begin{aligned}\rho_t + \nabla \cdot (\bar{a}\rho) &= \epsilon \Delta \rho + f_0(\rho, c), \\ c_t &= D \Delta c + g(\rho, c).\end{aligned}\tag{1}$$

Here $\rho(t, x)$ represent the density of a cell population and $c(t, x)$ concentrations or density of certain bio-chemicals. In addition, $\bar{a} = \sum_{i=1}^l f_i(c) \nabla c_i$. The functions f_i describe the strength and the sign of the tactic influence of each chemical c_i on the population density ρ . Two specific applications of this chemo-taxis model are tumour angiogenesis and pattern formation, which will be studied in this seminar.

The tumour angiogenesis model describes the process induced by the tumour which aims to establish a connection to the blood network (nutrient supply) in order to grow further.

Please derive a 1D model for tumour angiogenesis, and find out the meaning of the terms *advection*, *diffusion*, and *reaction*.

Some concerned materials, such as the paper by Chaplain & Stuart (1993) and the book by Hundsdorfer & Vener (part 1), can be download in the website

”<http://www.math.uni-konstanz.de/~yang>”