CONSERVATION LAWS AND HAMILTON-JACOBI EQUATIONS IN THE THEORY OF PLASTICITY

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Abstract. In standard models of plasticity the evolution equation for the plastic strain tensor, the constitutive equations, consist of a system of ordinary differential equations in time. As a result, the plastic deformation behavior shown by these models is independent of the size and dimensions of the plastic body. This contradicts experimental results, which show that the plastic yield limit is higher for thin films than for solid bodies with non-small dimensions in all directions.

In this talk we present a model for plasticity, in which the evolution equation for the plastic strain tensor is a first order partial differential equation, a Hamilton-Jacobi transport equation. One expects that for such a model the plastic behavior depends on the dimensions of the body.

The model has two forms, a "conservation law form" and a "Hamilton-Jacobi form". In the conservation law form the evolution equations consist of a system of conservation laws, whose solution is the dislocation density. The dislocation density can be equal to a "Dirac distribution supported on a curve"; such a distribution solution models a line dislocation in the crystal. The formulation of the system is chosen such that such line dislocations move with driving force given by the Peach-Köhler force. However, the idea is to consider less singular solutions of this system; these solutions can have jumps, along which Rankine-Hugoniot conditions must be satisfied.

The conservation law form of the model can be readily transformed to an equivalent second form, the Hamilton-Jacobi form, where it takes the form of a standard crystallographic model, however with the evolution equations for the strain tensor multiplied by the absolute value of the gradient of the internal variables. This is the form mentioned above, where the evolution equation is now a Hamilton-Jacobi transport equation.