LOCAL AND NONLOCAL PROBLEMS FOR STRONGLY HYPERBOLIC DIFFERENTIAL EQUATIONS IN BOUNDED DOMAINS

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Abstract. Very little is known yet concerning boundary problems for general partial differential equations. Although the local solvability of partial differential equations ia advanced enough at present, the questions of the existence of well posed boundary problems for sufficiently broad classes of differential operators attracted much less the attention. This is especially true to partial differential equations in bounded domains where (with the exception of elliptic equations) a list of well studied boundary problems seems to be sporadic (the Cauchy and the Goursat problems in characteristic cones, the mixed problem for hyperbolic operators; the latter problem for parabolic operators; the Tricomi problem, and some other tasks). It is clear that at present the general problem of assigning to each differential operator P a domain $D \subset \mathbb{R}^n$ and an additional operator B such that the problem

$$Pu = f \quad \text{in } D, \qquad Bu = g \quad \text{on } \mathcal{M} \subset \overline{D}, \tag{1}$$

is well posed, appears to be utopian dream. However, advances in the study of such a problem in the framework of some broad classes of operators does not look hopeless. The present talk relates to this very circle of problems.

In the space $\mathbb{R}^2$ we consider an arbitrary strictly hyperbolic differential operator of third order. To this operator we canonically assign domains D_1 and D_2 of two different types, and relate these domains to corresponding (quasi)boundary operators B_1 and B_2, respectively. As the first result we formulate some topological restrictions on the domains D_j and curves $\mathcal{M}$ under which the corresponding problem (1) is uniquely solvable in the scale of the spaces C^k. It is remarkable that one of these problems breaks two folklore taboos: *one* boundary condition on the *entire boundary* ∂D for a *hyperbolic* equation of *third order* defines a *well-posed* boundary problem. What is worth a special emphasizing is that instead of traditional (in the theory of boundary problems) integral and pseudodifferential equations the main technical role in the study of the problems in question play *functional* equations (which never arose before in the long-developed theory of these equations). The same functional equations appear (unexpectedly!) also when studying some new problems in integral geometry.

The second part of the talk is devoted to nonlocal quasiboundary problems for general second order hyperbolic differential operators in a characteristic rectangle D. Although some different nonlocal problems have been successfully considered in connection with elliptic and parabolic operators, practically there was no advancing in the theory of hyperbolic operators. In the present talk two nonlocal quasiboundary problems for second order hyperbolic operator in D with constant coefficients are discussed. In both cases we formulate conditions ensuring the unique solvability for each of these problems and also the conditions of their fredholmness. We demonstrate the examples showing the sharpness of these results: a violation of any above mentioned conditions leads to the omission of the corresponding result.

The nonlocal part of the talk is based partly on the join work with Peter Paneah (to be published in Transactions of Moscow Mathematical Society).