BOUNDED H_∞-CALCULUS FOR HYPOELLIPTIC PSEUDODIFFERENTIAL OPERATORS

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ABSTRACT. A closed operator A in a Banach space X admits a bounded H_∞-calculus if, roughly speaking, one can define $f(A)$ as a bounded operator in X for all functions f which are holomorphic and bounded on a sectorial domain which is strictly contained in the right complex half plane.

This property received some attention in recent years as it has applications in the theory of nonlinear evolution equations. In particular, the existence of a bounded H_∞-calculus implies maximal L_p-regularity for the equation

$$\partial_t - Au = f, \qquad u(0) = 0,$$

i.e., for any right-hand side $f \in L_p((0,T), X)$ there exists a unique solution u which is as regular as one can expect, namely $u \in W_p^1((0,T), X) \cap L_p((0,T), \mathcal{D}(A))$. In combination with linearization techniques and fixed point arguments this property may be used to derive solvability results for nonlinear problems.

The investigation if A admits a bounded H_∞-calculus is closely connected with the analysis of the resolvent of A. I will discuss this in case $A = A(x, D)$ is a pseudodifferential operator in the standard Hörmander class, the Beals-Fefferman class, and the Weyl-Hörmander class. The main feature is to find a simple criteria (sectorial hypoellipticity) for $A(x, D)$ that ensures the existence of the resolvent on the complementary sectorial domain and then to make use of its pseudodifferential structure for defining $f(A)$ via a Dunford integral, integrating f against the resolvent.

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