Universität Konstanz
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Polynomial Optimization – Computer Project 3

This project is a continuation of Computer Project 2, of Exercise 3 of the Problem Sets 2 and 3, as well as of Exercise 2 of the Problem Sets 4 and 5.

Thanks to the hard work of Franz, Sepp and yourself, Seefraß has extended its business volume considerably during the last semester. The boss wants to hire a new person but he hardly could find anybody who values the Seefraß working environment as much as Franz and Sepp do. Fortunately, Franz and Sepp have a sister called Zenz. After long negotiations, the boss manages to hire Zenz as an additional co-worker. Since the former work schedules were all designed for two workers, you get a follow-up contract as IT consultant to adjust the software so that it produces schedules for three workers instead of two.

(a) Create a copy `franzseppzenz.m` of the MATLAB file `franzsepp.m`. Rewrite the code in `franzseppzenz.m` in such a way that scheduling on three instead of two workers is handled. To this end, modify the constraints of the LP formulation (and thus also of the Sherali-Adams-LP as well as everything else that could be necessary in your code). Keep in mind to adapt the list scheduling procedure as well.

(b) What results do you get if you apply the `list_schedule` algorithm to Sherali-Adams schedules? Find an instance (k, t, R) where the `list_schedule` algorithm applied to the Sherali-Adams schedule $x \in S^{\mathrm{SA}}_{k,t,R}$ obtained from your code does not yield any schedule $y \in S_{k,t,R}$.

Attention: The notations $S^{\mathrm{LP}}_{k,t,R}$, $S^{\mathrm{SA}}_{k,t,R}$ and $S_{k,t,R}$ now silently designate the set of (fractional) (Sherali-Adams) plans *for three workers* instead of two workers. We do not introduce them formally but it is completely analogous to the notions defined for two workers.)

Hint: It might be hard to find such an example by hand and thus it can be helpful to run a script that generates random matrices $R \in \{0,1\}^{k\times t}$ for some values of t. You might try with $t \in \{4,5,6,7,8\}$. To avoid trivial infeasibilities, R should encode an irreflexive antisymmetric relation (i.e., $R_{ij} = 1 \implies R_{ji} = 0$). You can for instance try strictly *lower*[1] triangular instances of R.

(c) Now that you found a counterexample, the Seefraß boss is mad at Sherali and Adams. Yet they defend their idea: They claim you just would have to apply the process from Exercise 1 on Sheet 4 (or from Part (e) of Computer Project 2) twice

[1]Or upper triangular matrices. Which is better depends on your implementation of the list schedule procedure (why?). For most people, lower triangular might be the better choice.

instead of once. More precisely, you should again replace each of the linear equations and inequalities that define $S_{k,t,R}^{\mathrm{LP}}$ by the $2kt$ constraints obtained by multiplying the constraint with each x_{is} and each $1 - x_{is}$. In this way you get again the well-known quadratic constraints to which you add again the constraints $x_{is}^2 = x_{is}$. Sherali and Adams say that all you would have to do differently is that you should not linearize them directly as you did before. Instead you should replace each of these quadratic equations and inequalities again by the $2kt$ many constraints that arise from multiplying it with each x_{is} and each $1 - x_{is}$. Now you have a huge bunch of cubic constraints that you now should linearize in order to get an LP relaxation of the POP from Exercise 3(c) on Problem Set 2. Sherali and Adams call fractional schedules that arise from feasible solutions of this relaxation *Sherali-Adams schedules of the second generation*.

Extend `franzseppzenz.m` so that it constructs and solves the Sherali-Adams LP of the second generation described above. On a voluntary basis, you can even create one or several new MATLAB function files that could for example contain a function that implements the multiplication with the x_{is} and each $1 - x_{is}$. In this way, you could even reiterate this process further in order to produce Sherli-Adams LP relaxations of higher generations (whose definition you can now easily guess).

(d) Now consider the matrix R you found in (b) with corresponding k, t for which you did not get a feasible $y \in S_{k,t,R}$ after list scheduling. Compute a Sherali-Adams schedule of the second generation and use again the `list schedule` procedure to obtain some integer schedule out of it. Does this work better? Can you still find a counterexample?

Warning: Depending on your computer, you might have to calculate from several minutes to up to an hour or even more. The LP is huge.

Due by Friday, Juli 12th, 2019 at noon. All files must be sent attached to an electronic mail to Alexander Taveira Blomenhofer [2].

[2] `http://www.math.uni-konstanz.de/~blomenhofer/`