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## On the Malle Conjecture and the Self-Twisted Cover

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Abstract: The Malle conjecture predicts that the number of Galois extensions of  $\mathbb{Q}$  of given group G and with discriminant bounded from above by a real number y > 0 grows at least like  $y^a$ , for some specific exponent a > 0. This is known for nilpotent groups and for a few other groups. I will present a method that proves it for  $S_n$ ,  $A_n$ , many simple groups, and more generally for all regular Galois groups over  $\mathbb{Q}$ . The constructed Galois extensions can further be prescribed some notable local conditions. The method uses a new version of Hilberts irreducibility theorem that counts the specialized extensions and not just the specialization points. An important ingredient is the self-twisted cover that we will introduce.