PROSEMINAR: POSITIVITÄT UND QUADRATISCHE FORMEN

1. **25th October** - Katrin Loher
Review of topics from linear algebra.
Reference: Section 9.2 Hoffmann, Kunze - Linear Algebra
2. **8th November** - Katrin Stocker
In this talk we will give the definitions of positive and non-negative forms on real and complex vector spaces. We will characterise the positive forms in terms of their matrix representation. We will define the principal minors of a matrix and use this definition to take the first step towards finding a more useful characterisation of positivity of a form.
Reference: pages 325-328 Hoffmann, Kunze - Linear Algebra
3. **15th November** - Zoe Graser
Following on from the previous talk, we will give a characterisation of positive forms in terms of the principal minors of their matrix representation. We will then compare the real and complex cases. We will give the definition of a positive/non-negative linear operator on an inner product space and explain the relationship between positive operators, positive forms and positive matrices.
Reference: pages 328-331 Hoffmann, Kunze - Linear Algebra
4. **22th November** - Martin Faigle
In this talk we review the definition of a bilinear form $\beta : V \times W \to K$, introduce the space of bilinear forms $\mathrm{Bil}(V, W)$ and show that this space is isomorphic to the space $\mathrm{Hom}(V, W^*)$. We will then define the matrix representation of a bilinear form.
Reference : pages 13-17 (up to end of proof of F5) Lorenz - Lineare Algebra II
5. **29th November** - Julia Kraechter
In this talk we will discuss change of basis for bilinear forms and congruence of matrices. We will then go on to prove a theorem about non-degenerate bilinear forms and endomorphisms, that is:
if $\beta : V \times W \to K$ is a non-degenerate bilinear form and $\gamma : V \times W \to K$ is an arbitrary bilinear form then there exists a unique endomorphism f of V such that

$$\beta(f(x), y) = \gamma(x, y) \text{ for all } x \in V \text{ and } y \in W.$$

Reference: pages 17-22 (from F6, up to F8) Lorenz - Lineare Algebra II

6. **6th December** - Jennifer Ihlow

Parallel to the concept of the adjoint of an endomorphism of a vector space with respect to an inner product, we will investigate the concept of the adjoint of an endomorphism of a vector space with respect to a non-degenerate bilinear form. We will then go on to define the orthogonal complement of a subspace with respect to a bilinear form.

Reference: pages 22-26 (from F8, up to F10) Lorenz - Lineare Algebra II

7. **13th December** - Marie Kracker

We will start this talk by proving a dimension formula for the orthogonal complement of subspace with respect to a bilinear form. We will then go on to discuss symmetric, skew-symmetric and symplectic bilinear forms.

Reference: pages 26-30 (from F10) Lorenz - Lineare Algebra II

8. **20th December** - Nikolai Kracker

In this talk we will introduce quadratic forms and explain their connection to symmetric bilinear forms.

Reference: pages 31-34 Lorenz - Lineare Algebra II

————-Christmas————-

9. **10th January** - Eugen Makarov

In this talk we will discuss isometries of quadratic spaces, orthogonal bases and diagonalisation of quadratic forms over fields not of characteristic 2.

Reference: pages 35-39 (up to end of point (3)) Lorenz - Lineare Algebra II

10. **17th January** - Alessa Kupferschmid

In this talk we will prove a theorem of Witt (Witt'sche Kürzungssatz) and Sylvester's theorem for quadratic forms over $\mathbb{R}$.

Reference: pages 42-45 (up to end of section 2) plus Satz 5 page 63 Lorenz - Lineare Algebra II

11. **24th January** - Tanja Rohweder und Lucas Heitele

This talk will discuss when diophantine equations of the form

$$aX^2 + bY^2 + cZ^2 = 0$$

have non-trivial integer solutions.

Reference: pages 273-275 plus exercises 25 and 26 Ireland Rosen - A classical introduction to modern number theory

12. **31st January** - Anna-Lena Trautwein und Andreas Jüttler

In this talk we will discuss the continued fraction expansion of a square free natural number N and its relationship to the solution of the Pell equation

$$x^2 - Ny^2 = 1.$$

Reference: Chapter 8 section 1 and necessary background material on continued fractions from chapter 6 Baker - A concise introduction to number theory or a German reference from Merlin

13. **7th February** - Patrick Michalski

In this talk we will show that every integer is a sum of 4 squares.

Reference: Either Baker - A concise introduction to number theory or a German reference from Merlin.

14. **14th February** - Sina Schmitt

In this talk we will show that a prime number is the sum of 2 squares if and only if it is of the form $4k + 1$ where k is an integer.

Reference: Either Baker - A concise introduction to number theory or a German reference from Merlin.