

p-adically Closed Fields

Prof. Dr. S. Kuhlmann

Mitarbeiter: Dr. Lorna Gregory

2-stündig, Di. 14.00-16.00 Uhr, Raum D301

We will follow the book “Formally p -adic Fields” by Prestel and Roquette, covering the first 5 chapters. All page numbers in the following refer to this book.

- (1) 18th Oct **Vorbereitung**
- (2) 25th Oct & 8th Nov **Introduction/Prerequisites** - Lorna & Samuel
- (3) 15th & 22nd Nov **p -valued fields, p -ramification index, residue degree, coarse valuation, core field and fundamental degree formula.**
- Michael (15/11) & Fabian (22/11)
- (4) 29nd Nov **p -adically closed fields and p -adic closures.** - Philipp
- (5) 6th Nov & 13th Dec **Relatively algebraically closed subfields of p -adically closed fields.** -Karl-Heinz (6/12) & Samuel (13/12)
- (6) 20th Dec **Radical Structure Theorem.** -Lorna (20/12)

← Christmas Break →

- (7) **Algebraic Embedding Theorem.**
- (8) **General Embedding Theorem.**
- (9) **Model theory of p -adically closed fields.** - Wolfgang

(3) **p -valued fields, p -ramification index, residue degree, the coarse valuation, the core field and the fundamental degree formula.**

In these seminars we will introduce the notion of a p -valued field and define the p -ramification index and residue degree. We will give examples of p -valued fields. We will define the coarse valuation and core field of a p -valued field. We will end these seminars by proving the fundamental degree formula (lemma 2.9) for finite extensions of p -valued fields with Henselian base field. (Pages 13-19 and 22-32, suggested split: Micheal (13-19 & 22-24), Fabian (25-32))

(4) **p -adically closed fields and p -adic closures.**

We will characterise p -adically closed fields as Henselian p -valued fields with value group a \mathbb{Z} -group. We will then go on to show that a p -valued field has unique (up to isomorphism) p -adic closure if and only if the value group is a \mathbb{Z} -group. (Pages 32-37)

(5) **Relatively algebraically closed subfields of p -adically closed fields.**

In these seminars we will show that a relatively algebraically closed subfield K of a p -adically closed field L is p -adically closed when endowed with the valuation induced by the valuation on L and that K is of the same p -rank as L . (Pages 38-47)

(6) **Radical Structure Theorem.**

In this seminar we will show that an algebraic extension of a Henselian p -valued field of the same p -rank is generated by radical elements. (Pages 48-53)

(7) Algebraic Embedding Theorem.

In this seminar, given an algebraic extension L/K of p -valued fields of equal p -rank and an arbitrary Henselian valued extension L' of K we will give a necessary and sufficient condition under which L is K -isomorphically embeddable into L' as valued fields. We will then go on to give a description of the lattice of intermediate fields between a Henselian p -valued field and a p -adic closure of K based on the value group of K . (Pages 54-61)

(8) The General Embedding Theorem.

In these seminars we will prove the General Embedding Theorem for p -valued fields of p -rank d . This is the general version of the Algebraic Embedding Theorem where L' is now assumed to be κ -saturated for $\kappa > |L|$ and L is no longer necessarily an algebraic extension of K . (Chapter 4, pages 62-71, 71-76 and 77-82)

(9) Model theory of p -adically closed fields.

In these seminars we will give an axiomatisation of the class of p -valued fields and the class of p -adically closed fields of p -rank d in the language of valued fields. We will show that the theory of p -adically closed fields of p -rank d is model complete, decidable and has quantifier elimination in the language of valued fields extended by unary predicates for n th powers and $d - 1$ constant symbols. (Pages 83-91)